Burwell Village College Primary

Calculation Policy

Introduction

This policy is intended to help all staff in understanding the sequencing of teaching and progression of methods relating to the four rules of calculation – addition, subtraction, multiplication and division. It is based upon the requirements and expectations laid out in the National Curriculum.

This policy promotes the learning of concrete, pictorial and abstract methods. It is intended that this policy will allow consistency across the school in teaching calculation of the four operations, aiding children's progression in learning and transition between classes and key stages.

Teachers will need to be flexible in their approach to teaching the methods for solving the four operations, recognising when children are ready to progress to the next step whilst other children will need consolidation of the current step.

Aims of the Policy:

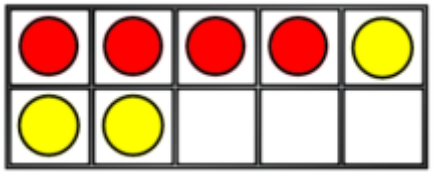
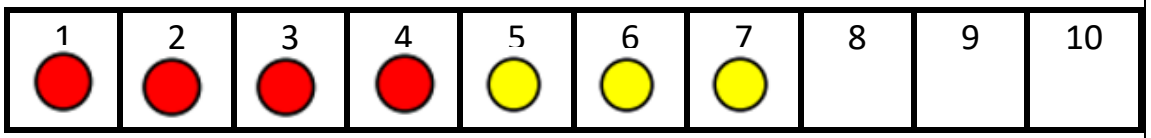
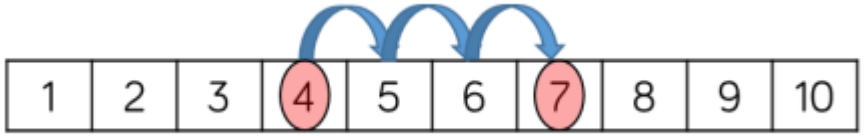
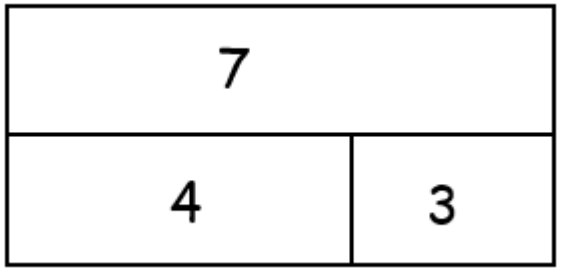
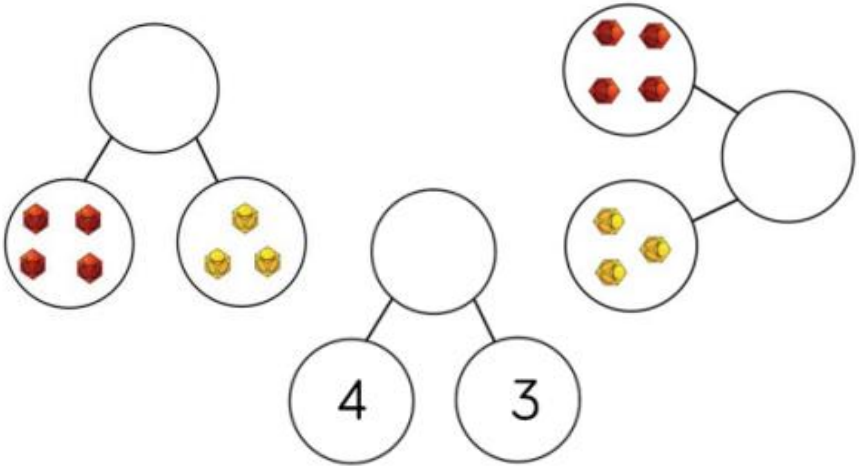
- To ensure consistency within teaching across the school
- To ensure children have a strong understanding of various calculation methods
- To ensure children can use these methods accurately and confidently
- To develop children's fluency with calculations
- To support teachers in helping deepen understanding through the use of concrete manipulatives and pictorial representations
- To model the strategies used in school for parents and all staff

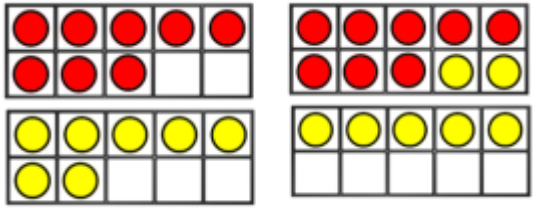
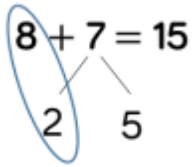
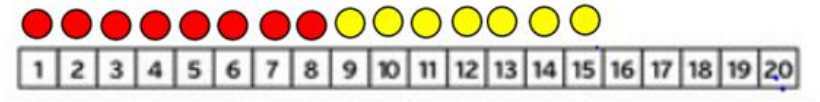
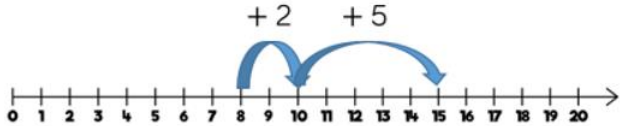
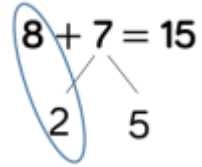
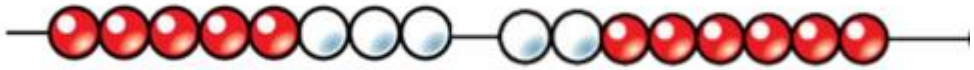
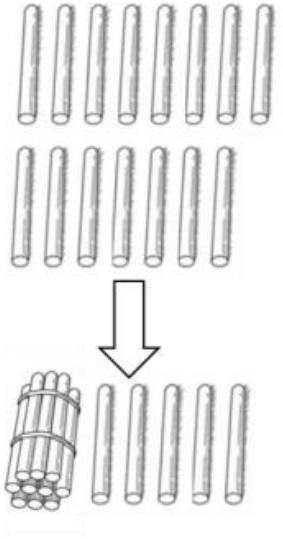
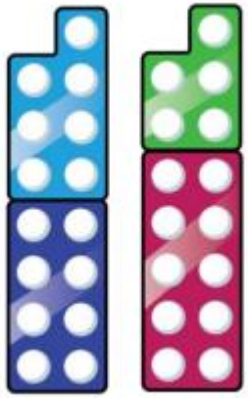
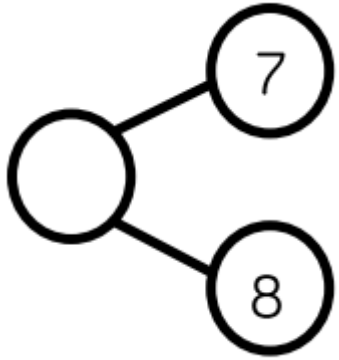
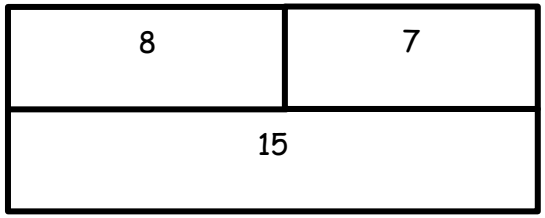
Mathematical Language

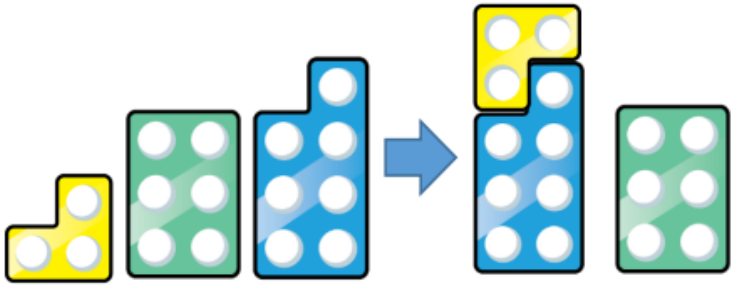
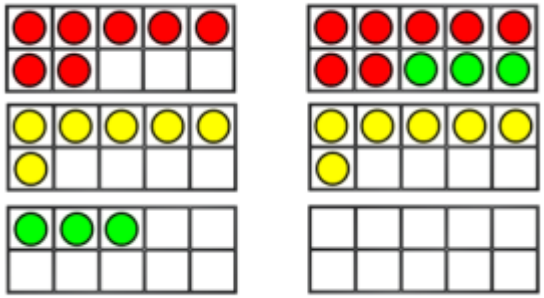
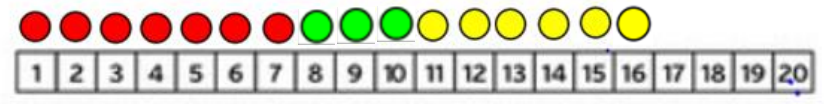
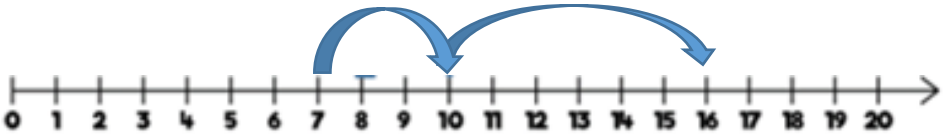
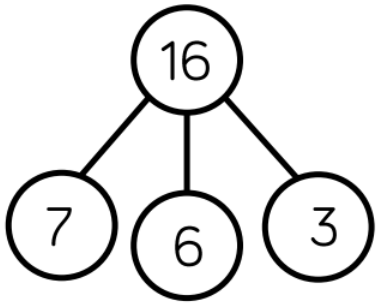
We value the importance of children using the correct mathematical language. As a school, we expect the use of subject-specific terminology to be used. It is essential that teachers refer to and use the vocabulary appropriate for each lesson to ensure appropriate mathematical vocabulary is consistently used. New vocabulary should be explained within lessons, and high expectations of using this language is essential.

Please see document at the end of the policy for further mathematical language guidance.

Progression in teaching addition:

Small step/stage	Models and representations
Add two 1-digit numbers within 10 including number bonds to 10	        Children to see calculations in a variety of models and then apply this to number tracks. Note: part whole models to be shown in different variations and bar models to vary with parts at the top and whole at the bottom.

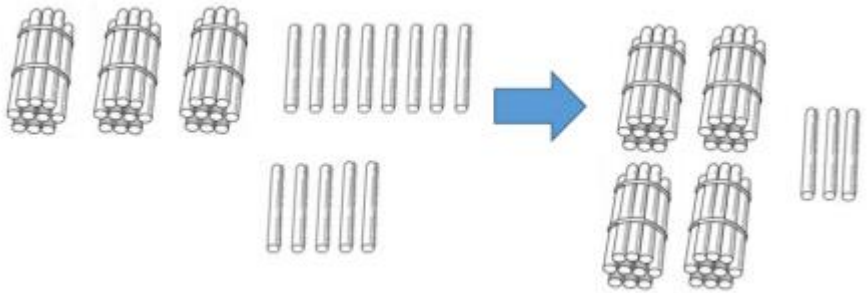
Add 1 and 2-digit numbers within 20	  Children need to see how numbers can be partitioned to make 10 so that they can use the number line effectively.     Different manipulatives can be used to represent the exchange.    

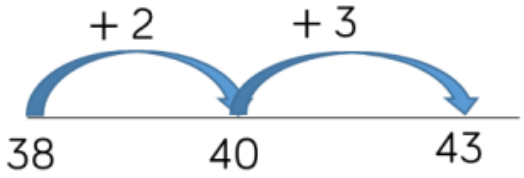
Add three 1-digit numbers	Children should be encouraged to look for number bonds to 10 or doubles to add the numbers more efficiently. $7 + 6 + 3 = 16$ 10       I bought a packet of crisps, a chocolate bar and a cupcake. How much did I spend?

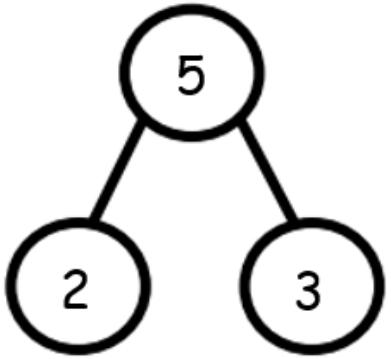
Add 1 and 2-digit numbers to 100	Children use a number line to count in ones from the larger number. Next, introduce models and representations that allow children to apply their knowledge of number bonds in order to calculate more efficiently.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Straws, dienes and number squares can support children to find number bonds.

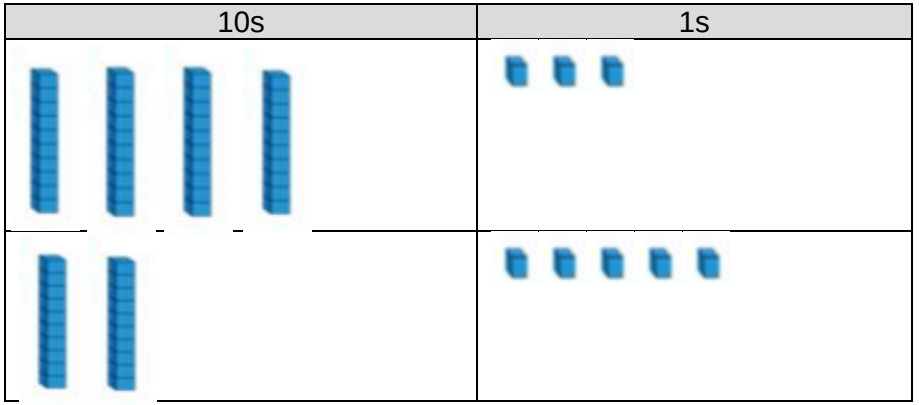


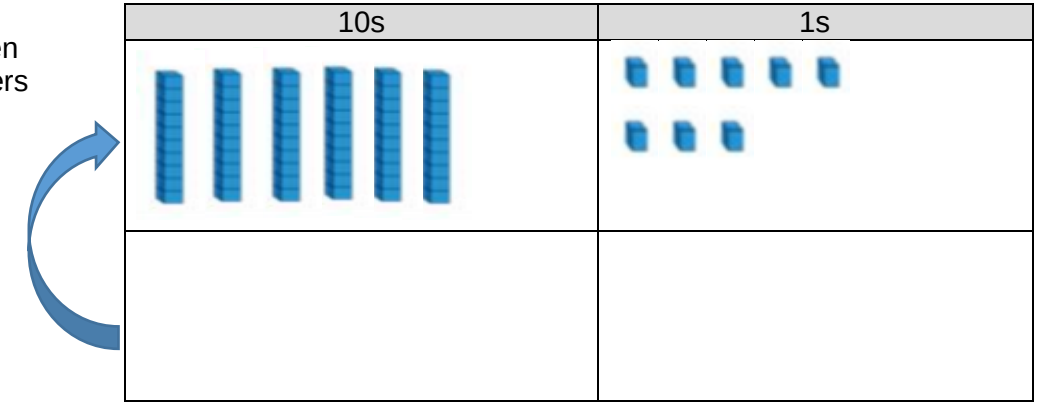


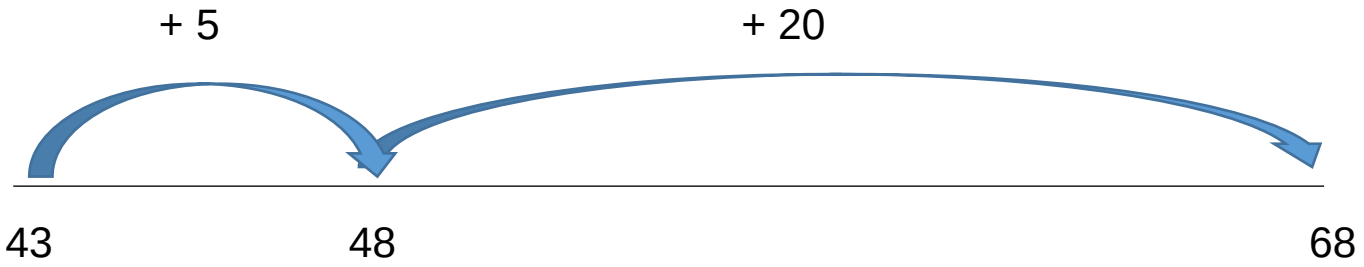


38 + 5 = 43

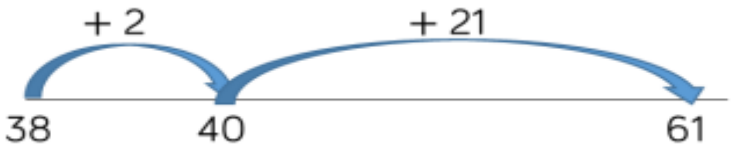
Add a pair (or more) of two-digit numbers, without exchanging	Encourage children to reason when adding without an exchange. 43 + 25 = 68 Look at the 1s column. What do you notice? 3 + 5 = 8 Look at the 10s column. What do you notice? 40 + 20 = 60 When using the place value grid, children add the ones first by bringing the counters **up** into the top box and then the tens. Blank number lines also used to add ones and then tens. Children should recognise that once they add the ones, the ones column will not change.

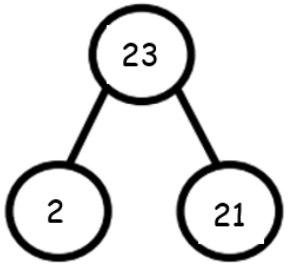




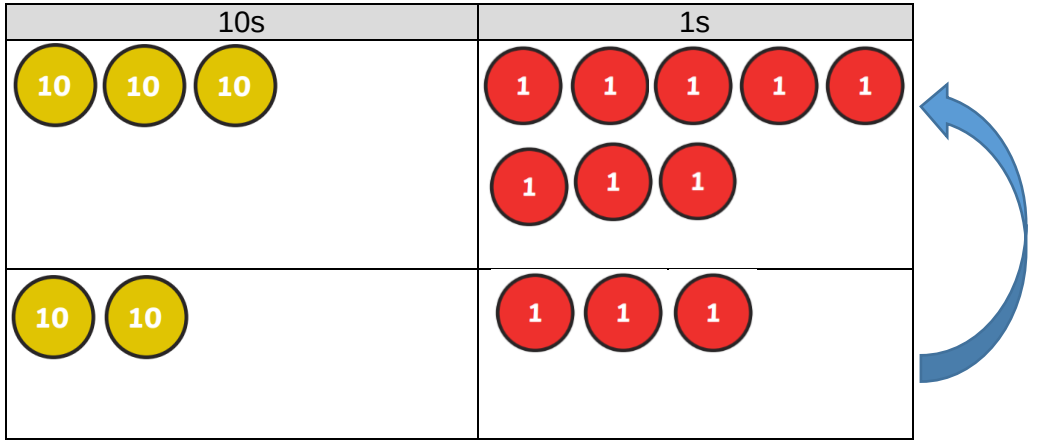


Add a pair (or more) of two-digit numbers, exchanging ones	Before introducing the formal method, children are to use a blank number line to jump to multiples of 10 to become more efficient. Children should also reason that 8 plus 3 is more than 10 therefore the ones and tens columns will change. The part-whole model is to be used to show number bonds.

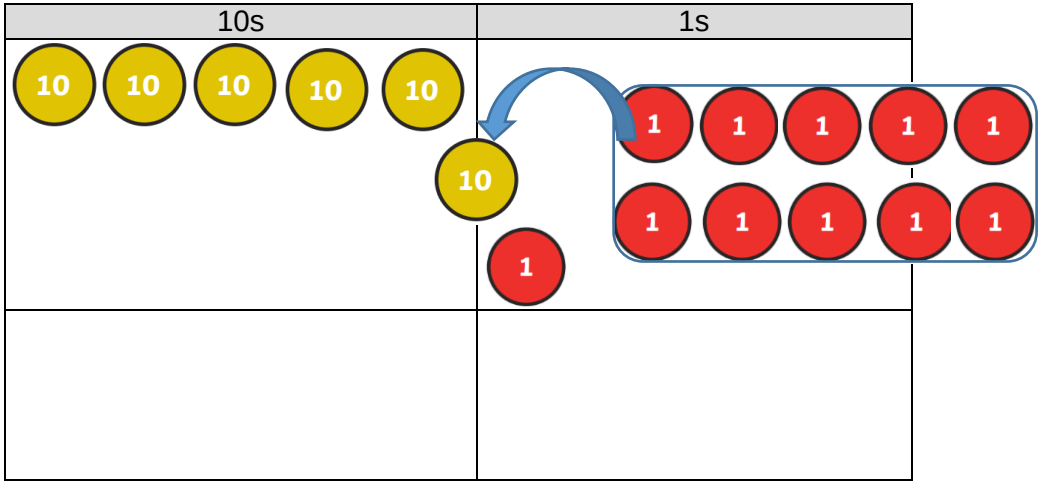




38 + 23 = 61



Move counters or dienes **up** into the top box of the place value chart.

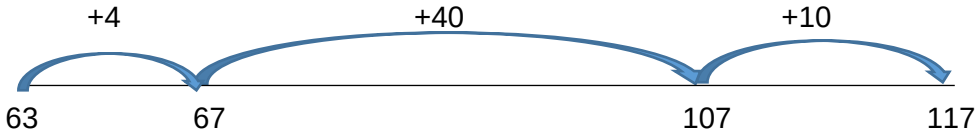


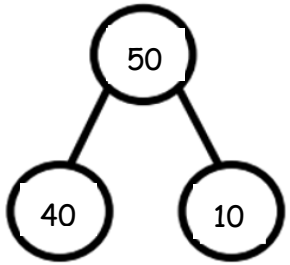
Model exchanging by taking the 10 ones **off** the chart and replacing with a ten into the 10s column.

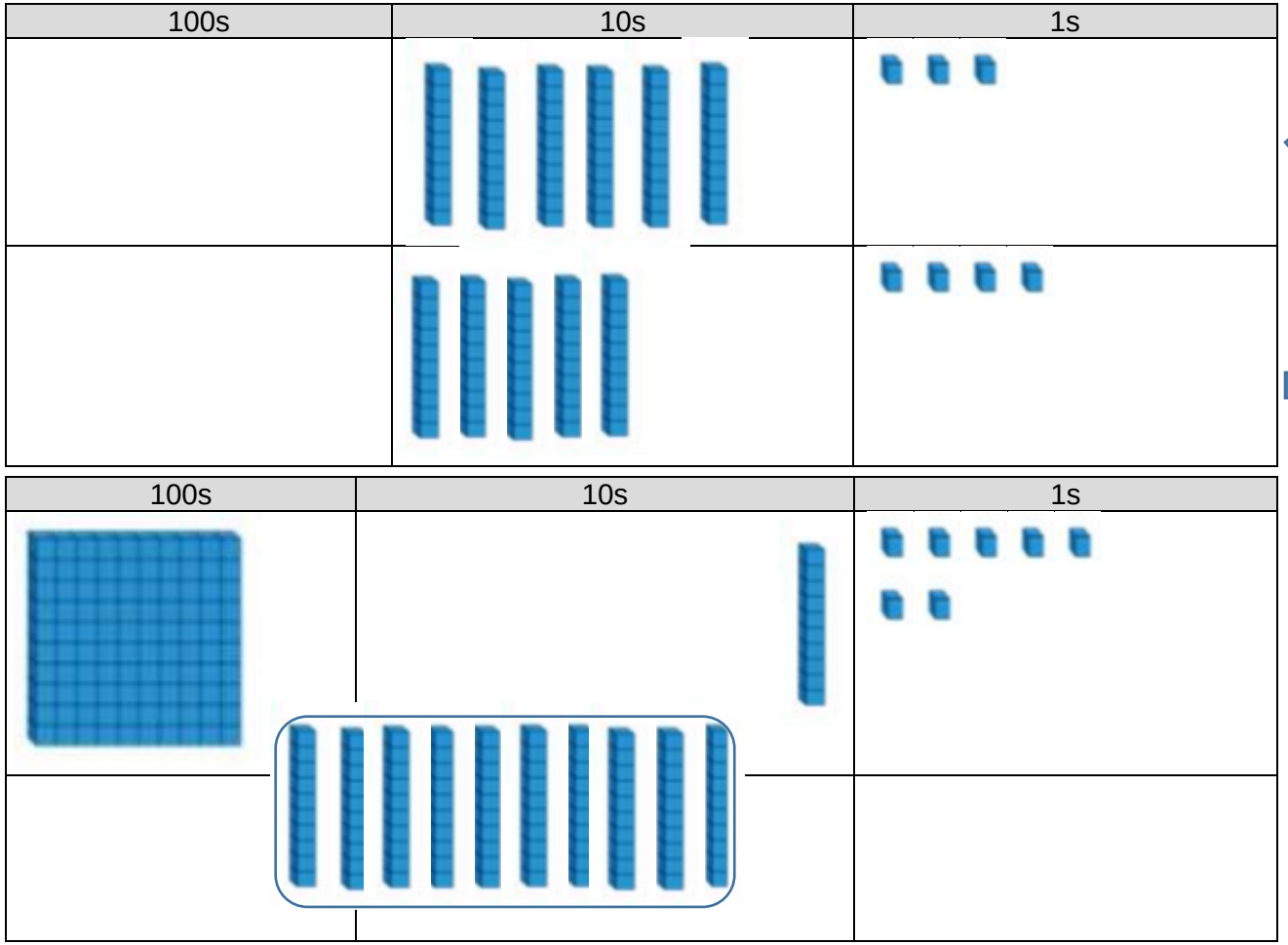
$$\begin{array}{r} 38 \\ +\ 23 \\ \hline 61 \\ \hline 1 \end{array}$$

Record the formal method next to the concrete model.

Add a pair (or more) of two-digit numbers, exchanging tens	Before using the formal method, children are to use a blank number line to jump to multiples of 10 to become more efficient. Children should also reason that 60 plus 50 is more than 100 therefore the tens and hundreds columns will change. The part-whole model is to be used to show number bonds.







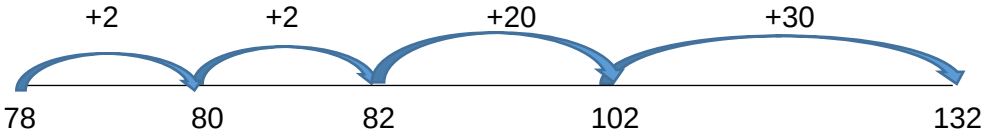
Move counters or dienes **up** into the top box of the place value chart.

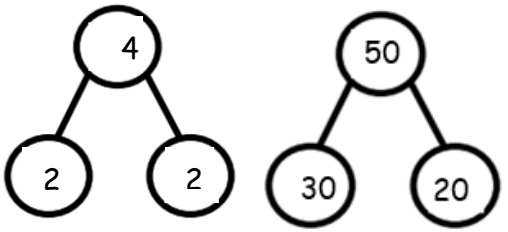
Model exchanging by taking the 10 tens **off** the chart and replacing with a hundred in the 100s column.

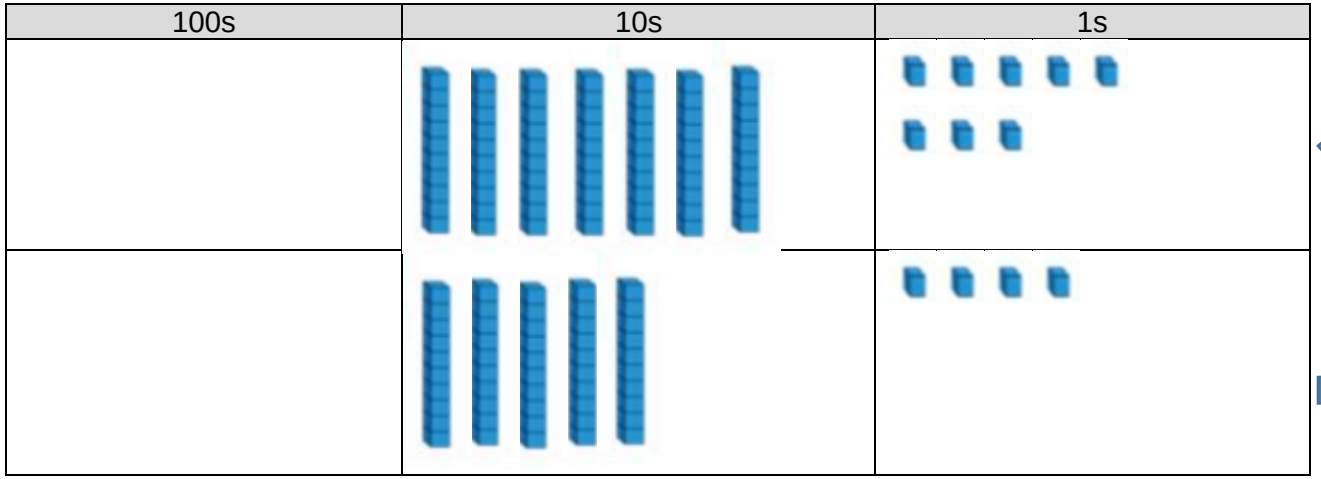
$$\begin{array}{r} 63 \\ +\,54 \\ \hline 117 \\ \hline 1 \end{array}$$

Record the formal method next to the concrete model.

Add a pair (or more) of two-digit numbers, exchanging ones and tens	Before using the formal method, children are to use a blank number line to jump to multiples of 10 to become more efficient. Children should also reason that 8 plus 4 is more than 10 therefore the ones and tens columns will change and that 70 + 50 is more than 100 so the hundreds column will change also.

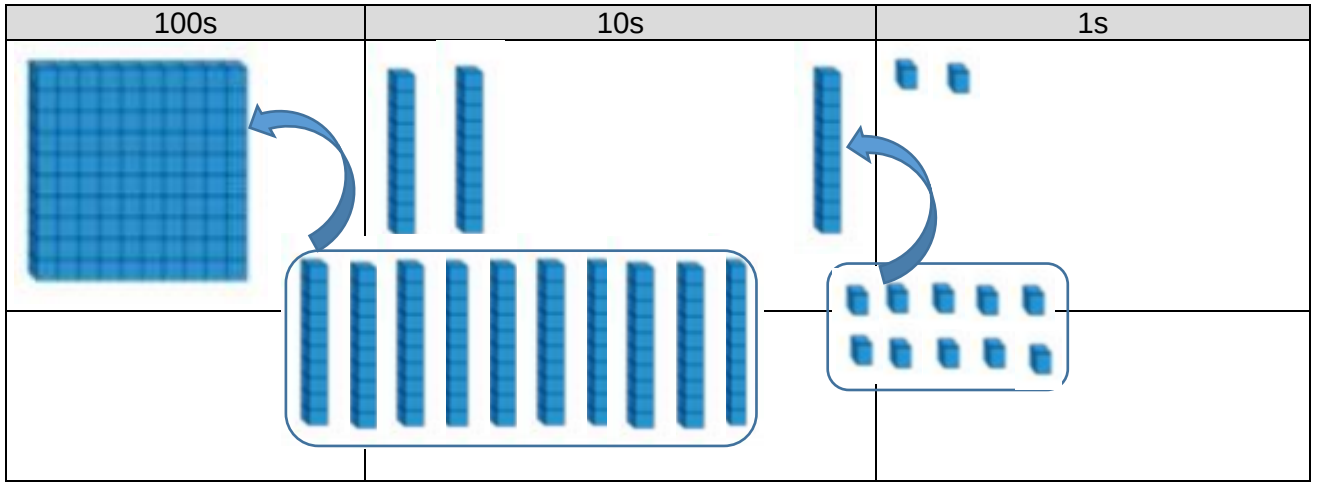






Move counters or dienes **up** into the top box of the place value chart.

Model exchanging by taking the 10 tens **off** the chart and replacing with a hundred in the 100s column.

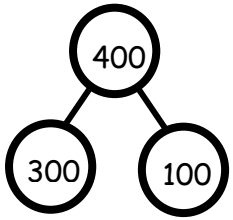


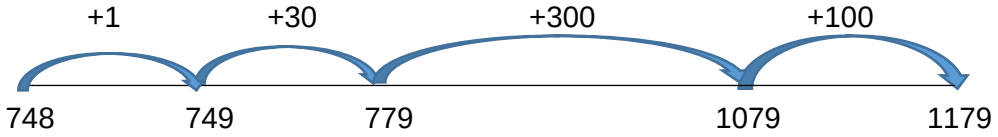
$$\begin{array}{r} 78 \\ +\,54 \\ \hline 132 \\ \hline 1\ 1 \end{array}$$

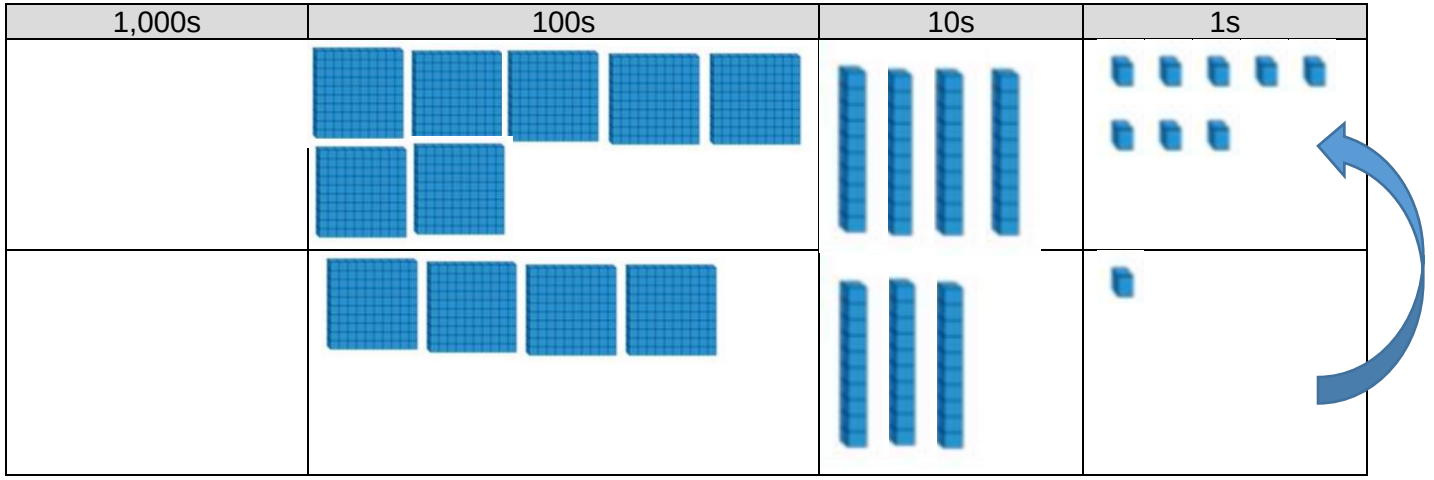
Record the formal method next to the concrete model.

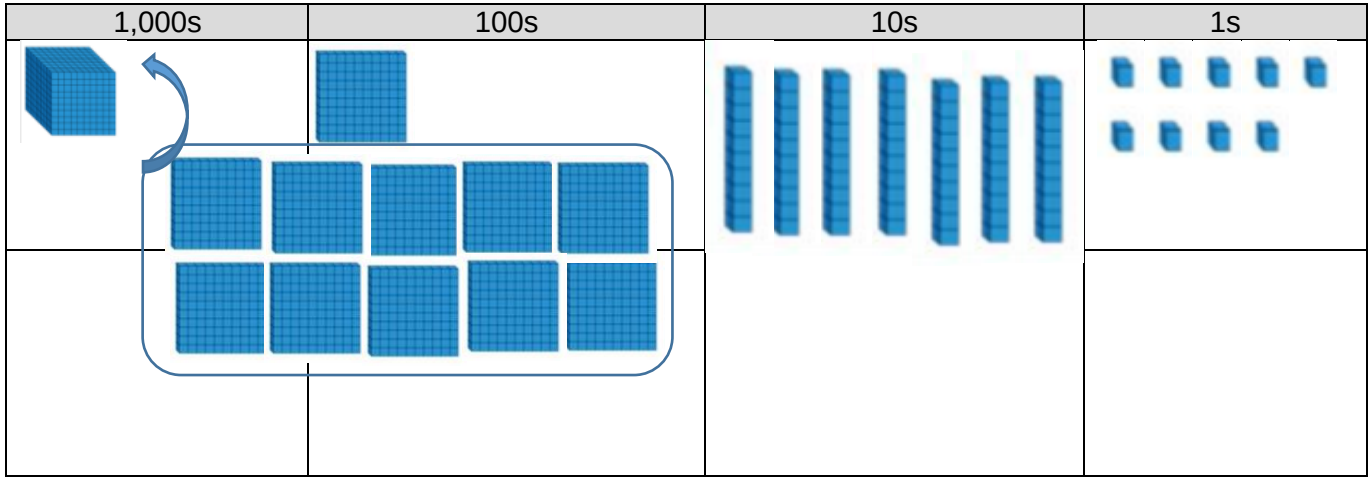
Add a pair (or more) of three-digit numbers or a three-digit number to a two-digit number, exchanging hundreds

Before using the formal method, children are to use a blank number line to become more efficient. Children should also reason that 8 plus 1 is less than 10 therefore only the ones column will change first, and that 40 + 30 is less than 100 so the tens column will change next. However, they should see that 700 add 400 is more than 1,000.









Move counters or dienes **up** into the top box of the place value chart.

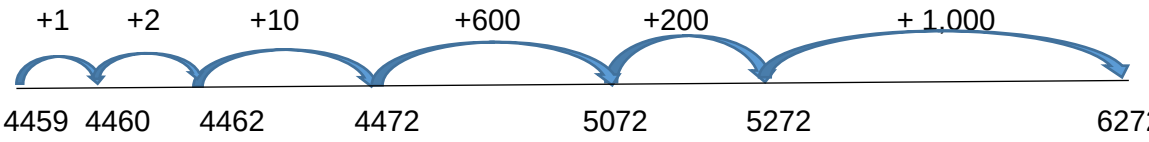
Model **exchanging** by taking the 10 hundreds **off** the chart and replacing with a thousand in the 1,000s column.

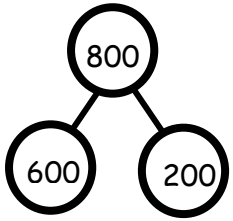
$$\begin{array}{r} 748 \\ +\,431 \\ \hline 1179 \\ \hline 1 \end{array}$$

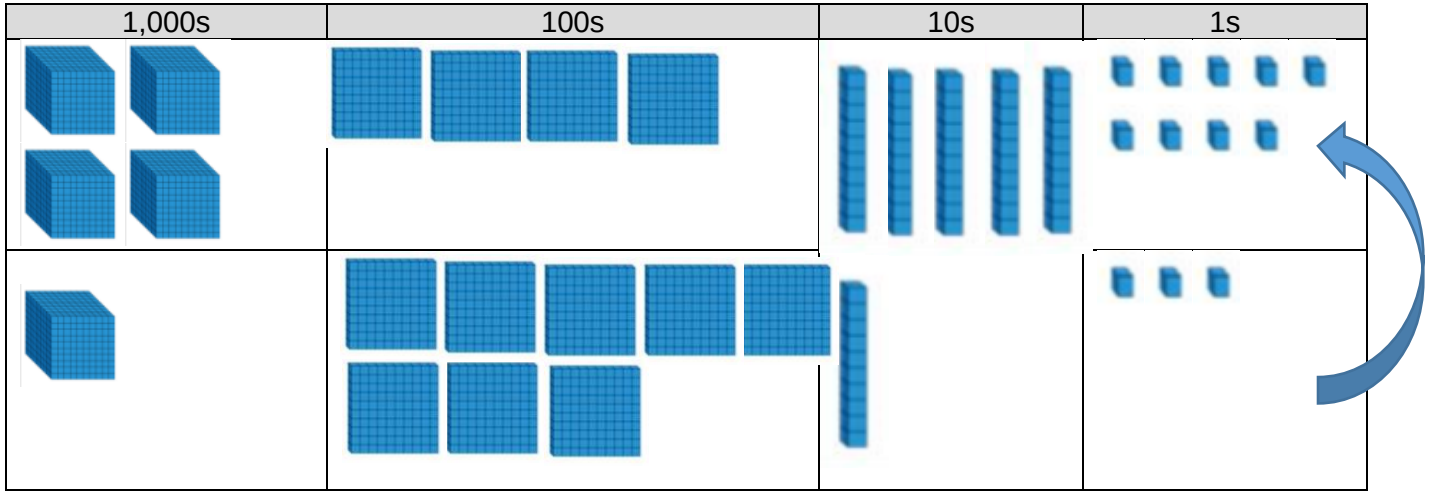
Record the formal method next to the concrete model.

Add a pair (or more) of four digit or greater numbers together, using exchanging across some or all place values

Before using the formal method, children are to use a blank number line to become more efficient. Children should also be able to use their reasoning skills to check if their solution is reasonable.

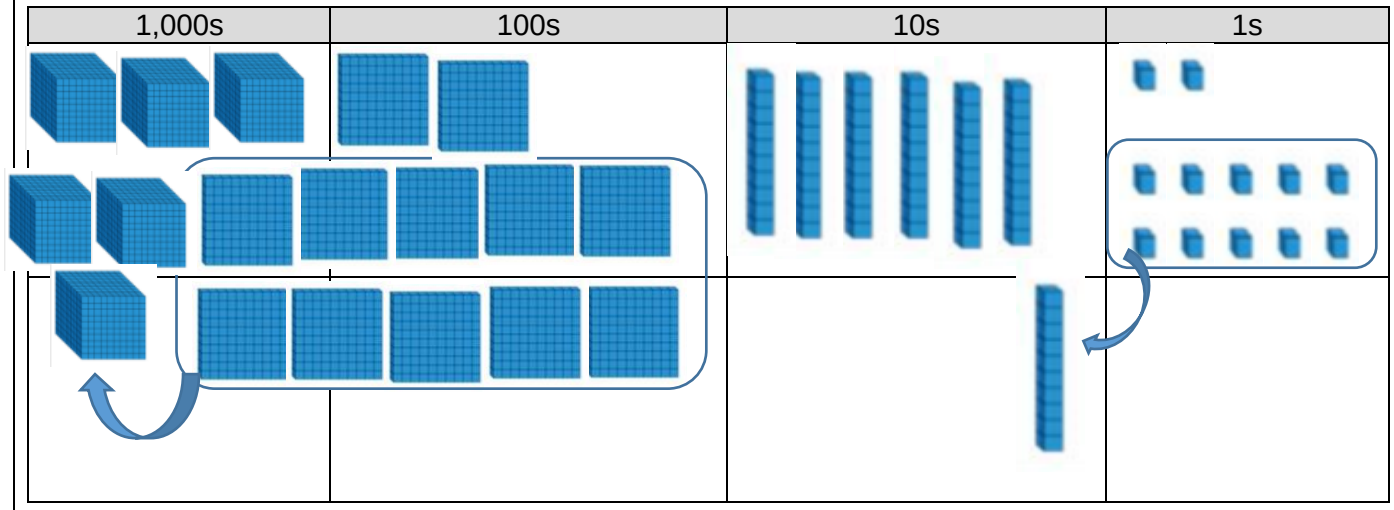






Move counters or dienes **up** into the top box of the place value chart.

Model **exchanging** by taking the 10 ones **off** the chart and replacing with a ten in the 10s column. Repeat with the hundreds.

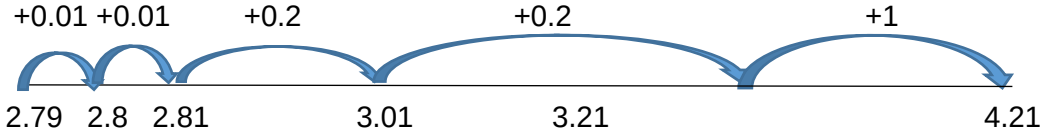


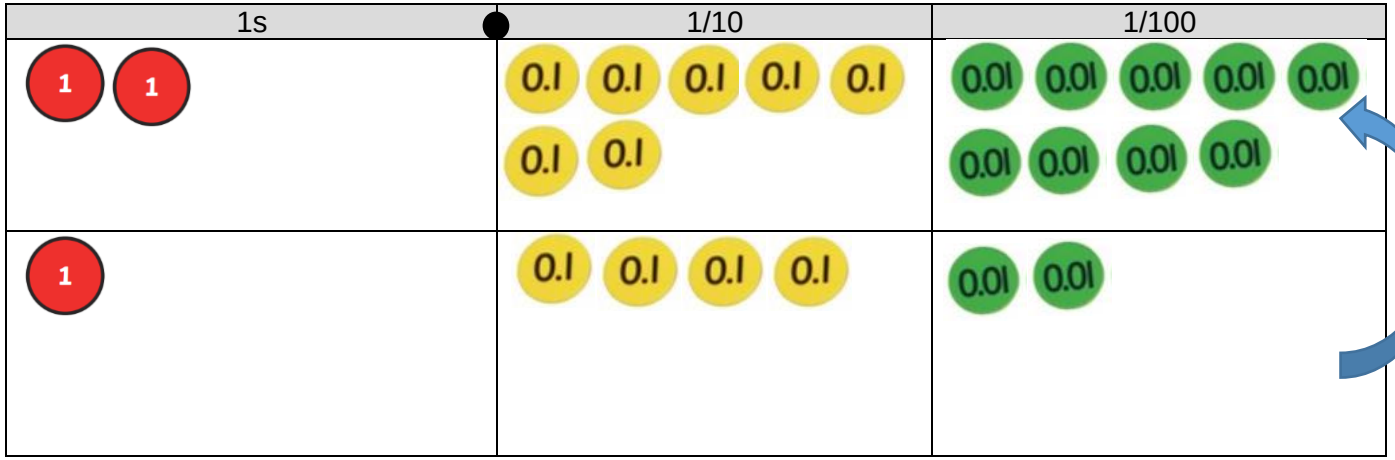
$$\begin{array}{r} 4459 \\ +\,1813 \\ \hline 6272 \\ 1\quad 1 \end{array}$$

Record the formal method next to the concrete model.

Add a pair (or more) of decimals together, ensuring digits lined up in place values, with exchanging between decimal values and across the decimal point to ones

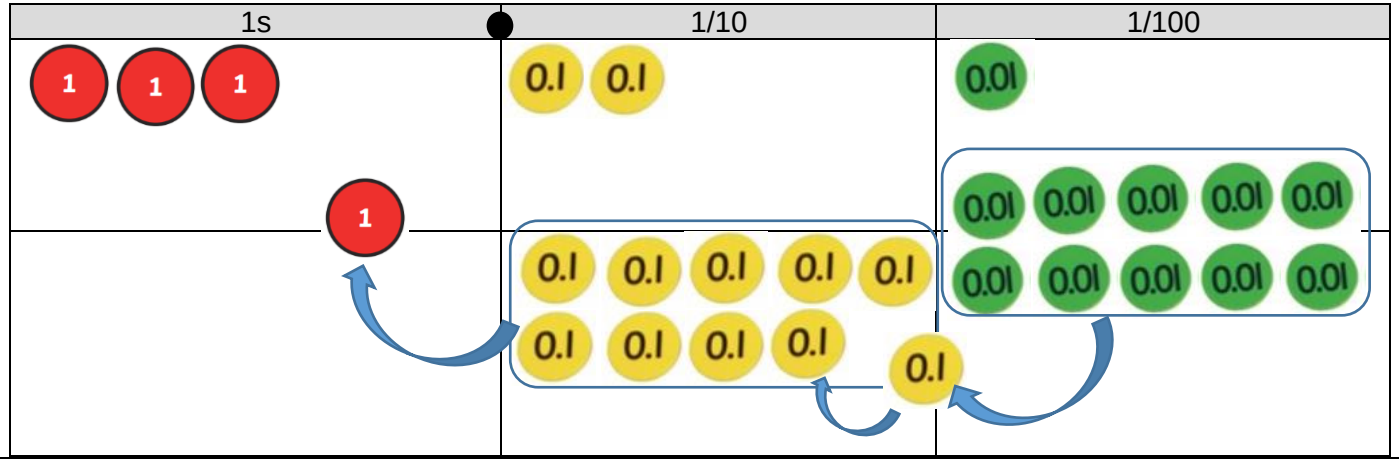
Before using the formal method, children are to use a blank number line to become more efficient. Children should also be able to use their reasoning skills to check if their solution is reasonable.





Move counters of dienes **up** into the top box of the place value chart.

Model exchanging by taking the 10 hundredths **off** the chart and replacing with a 1/10 in the 1/10 column. Repeat with tenths.



$$\begin{array}{r} 2.79 \\ +1.42 \\ \hline 4.21 \\ \scriptstyle 1 \quad 1 \end{array}$$

Record the formal method next to the concrete model.

Throughout all teaching of written methods for addition, children need to be given time to practise and consolidate skills and must be given opportunities to apply these written methods, at whatever stage they may be at to solving real-life problems, in the context of measures and money, and within the other strands of mathematics.

ADDITION NATIONAL CURRICULUM CALCULATION GUIDANCE

Year 1 pupils should be taught to:

- read, write and interpret mathematical statements involving addition (+) and equals (=) signs
- represent and use number bonds up to 20
- add one-digit and two-digit numbers to 20, including zero
- solve one-step problems that involve addition, using concrete objects and pictorial representations, and missing number problems

Year 2 pupils should be taught to:

- solve problems with addition using concrete objects and pictorial representations, including those involving numbers, quantities and measures
- apply their increasing knowledge of mental and written methods
- recall and use addition facts to 20 fluently, and derive and use related facts up to 100
- add numbers using concrete objects, pictorial representations, and mentally, including:
    - a two-digit number and ones
    - a two-digit number and tens
    - two two-digit numbers
    - adding three one-digit numbers
- show that addition of two numbers can be done in any order (commutative)
- recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems

Year 3 pupils should be taught to:

- add numbers mentally, including:
    - a three-digit number and ones
    - a three-digit number and tens
    - a three-digit number and hundreds
- add numbers with up to three digits, using formal written methods of columnar addition
- estimate the answer to a calculation and use inverse operations to check answers

- solve problems, including missing number problems, using number facts, place value, and more complex addition

Year 4 pupils should be taught to:

- add numbers with up to four digits using the formal written methods of columnar addition
- add decimal numbers up to two decimal places (in the context of money and measures)
- estimate and use inverse operations to check answers to a calculation
- solve addition two-step problems in contexts, deciding which operations and methods to use and why

Year 5 pupils should be taught to:

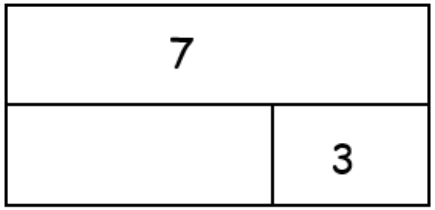
- add whole numbers with more than four digits, using formal written methods of columnar addition
- add decimal numbers with more than two decimal places
- add numbers mentally with increasingly large numbers
- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- solve addition multi-step problems in contexts, deciding which operations and methods to use and why

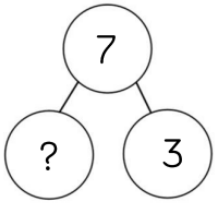
Year 6 pupils should be taught to:

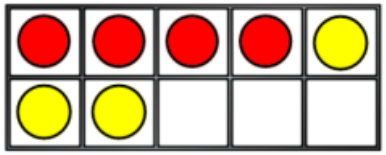
- perform mental calculations, including with mixed operations and large numbers
- use their knowledge of the order of operations to carry out calculations involving the four operations
- solve addition multi-step problems in contexts, deciding which operations and methods to use and why
- solve problems involving addition
- use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy

Progression in teaching subtraction:

Small step/stage	Models and representations
Subtract two 1-digit numbers to 10	

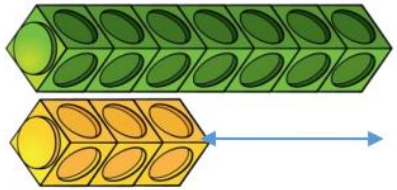




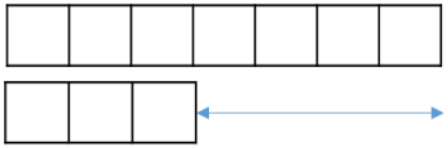


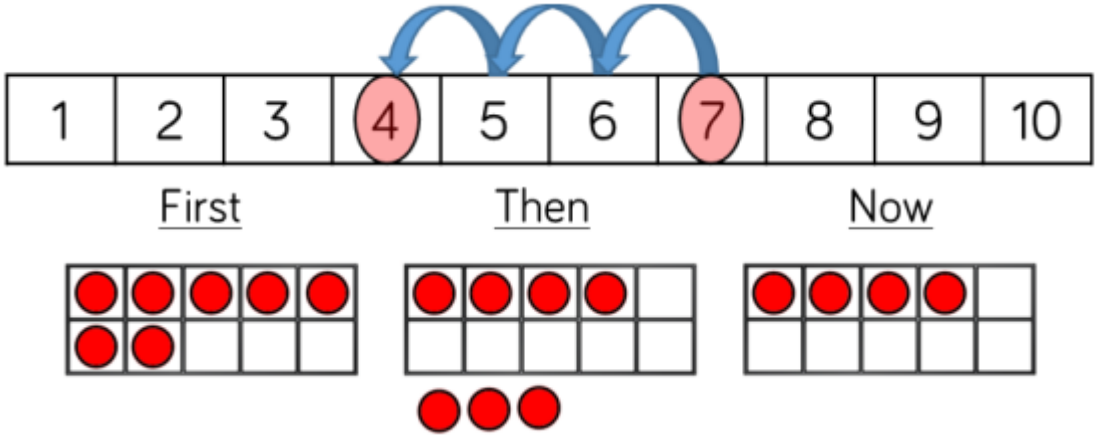
Tens frames, number tracks, bead strings and single bar models support reduction.

Part- whole models, bar models, tens frames and number shapes support partitioning.

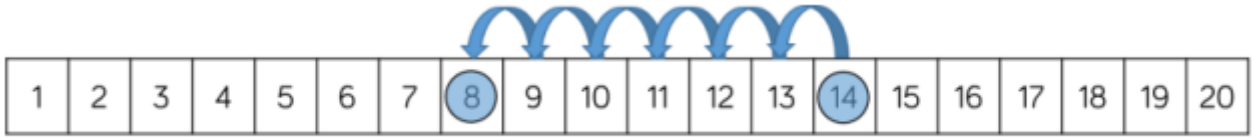
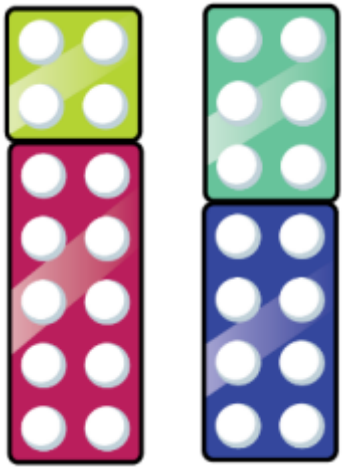
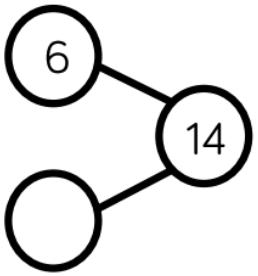
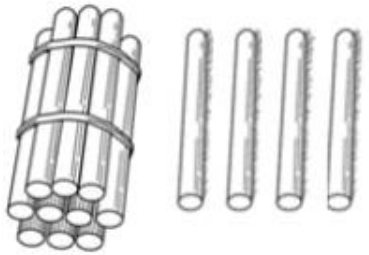
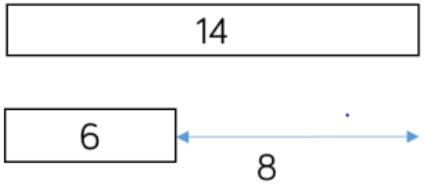
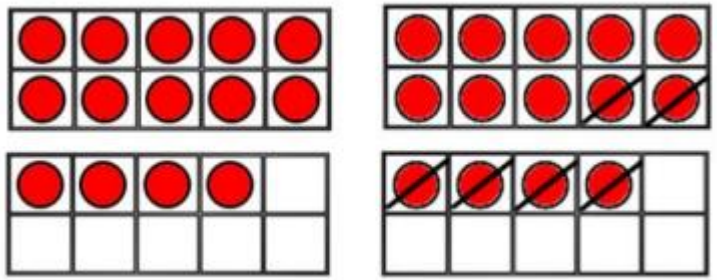
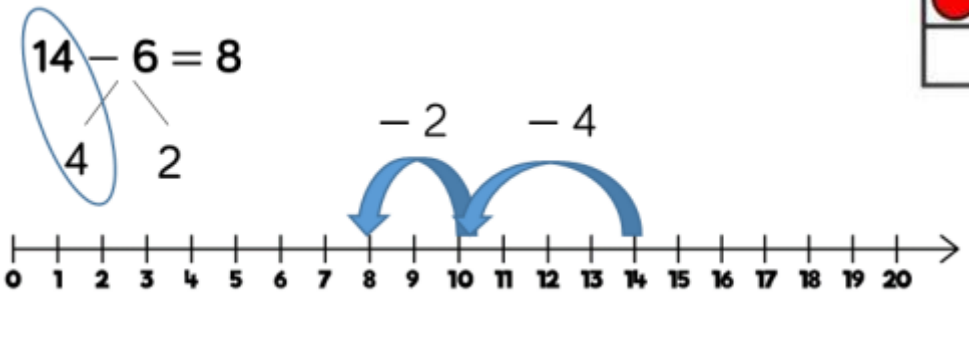
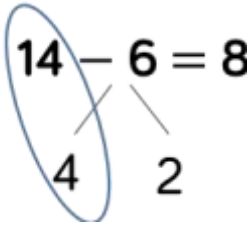


Cubes and bar models with two parts support finding the difference.

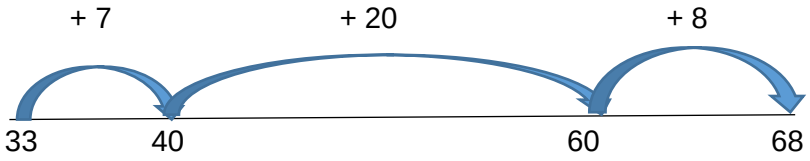




Use first…, then… and now… sentence stems to create stories to support children's understanding.

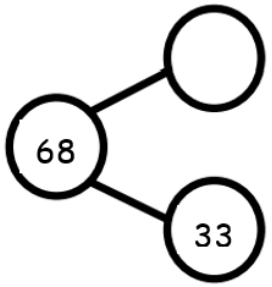
Subtract 1 and 2-digit numbers to 20	  When subtracting one-digit numbers that cross 10, It is important to highlight the importance of ten ones being the same as one ten. Children should be encouraged to find the number bond to 10 when partitioning the subtracted number. Tens frames, number shapes and number lines are great for representing this       

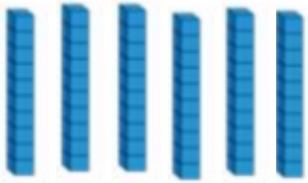
Subtract a single-digit or two-digit number from a two-digit number, without exchanging



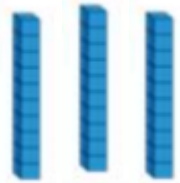
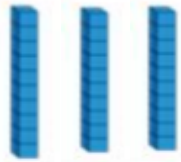
68 – 33 = 35

Children now use the number line to count on.



10s	1s
	

68	
33	

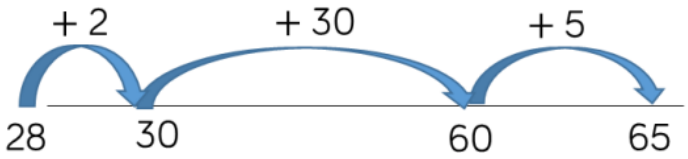
10s	1s
	
	

$$\begin{array}{r} 68 \\ -\,33 \\ \hline 35 \end{array} \qquad \begin{array}{r} 35 \\ +33 \\ \hline 68 \end{array}$$

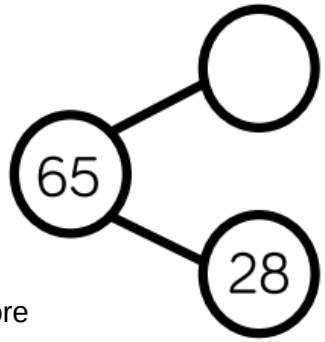
When showing the inverse, the answer goes at the top so that it matches the representation.

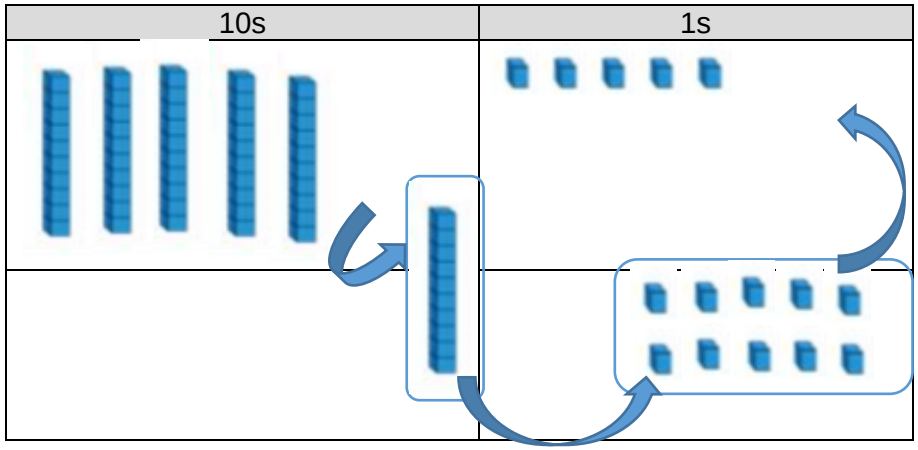
When using the place value chart, counters move **down** so that children can see the two parts. Putting the counters **up again** (add) will allow children to see the relationship between the **inverse** operation.

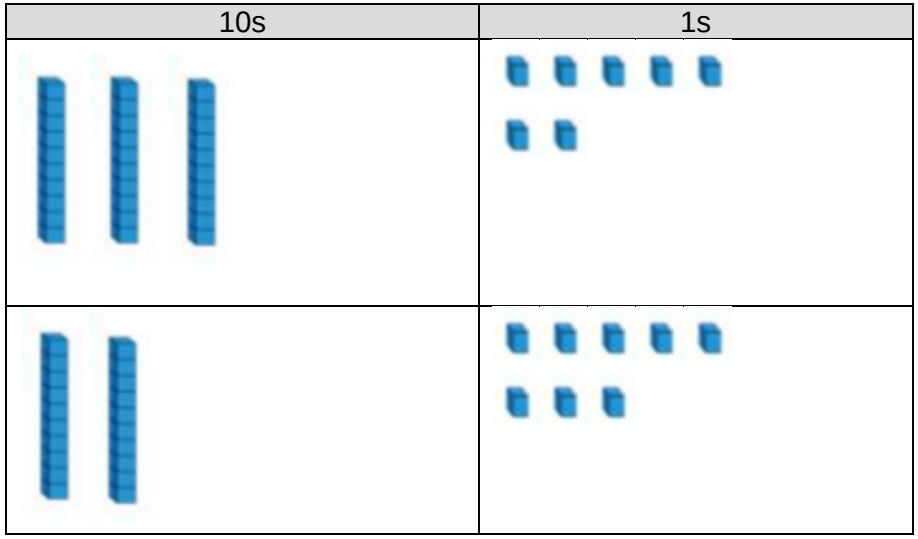
Subtract a single-digit or two-digit number from a two-digit number, exchanging from tens



28	?
65	







Children should be encouraged to reason before calculating. They should recognise that there are not enough ones and that an exchange will take place.

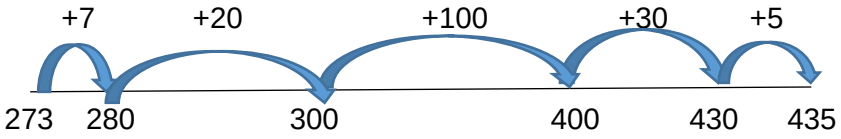
8 cannot be subtracted from 5 so one ten is **exchanged** for ten ones. The ten must come **off** before the ones go **on**.

$$\begin{array}{r} {}^{5}\cancel{6}\,{}^{1}5 \\ -\,2\,8 \\ \hline 3\,7 \end{array} \qquad \begin{array}{r} 3\,7 \\ +2\,8 \\ \hline 6\,5 \\ {\scriptstyle 1} \end{array}$$

Ensure children write the formal calculation alongside the concrete model so that they can see the links to the written method and the inverse operation (move the dienes back up).

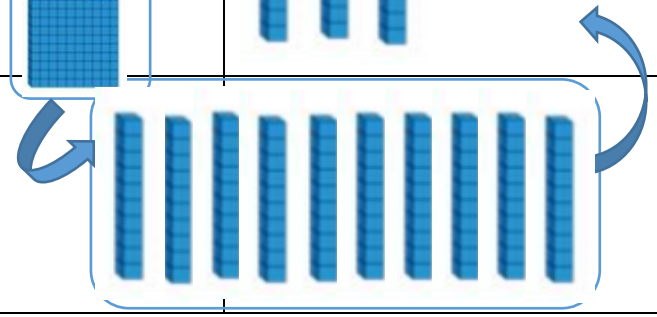
Subtract a two-digit or three-digit number from a three-digit number, exchanging from hundreds

435 – 273 = 162



When subtracting 70 from 30, we don't have enough tens so one hundred is **exchanged** for ten tens. The hundred must come **off** before the tens go **on**. Again, children should be encouraged to reason and recognise an exchange is needed before calculating.

100s	10s	1s



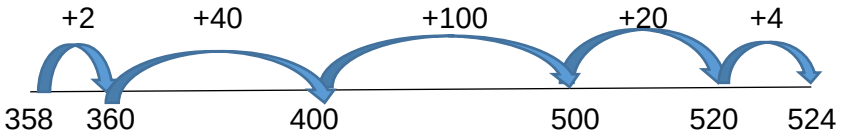
100s	10s	1s

$$\begin{array}{r} {}^{3}\not{4}\,{}^{1}\not{3}5 \\ -\,273 \\ \hline 162 \end{array}$$

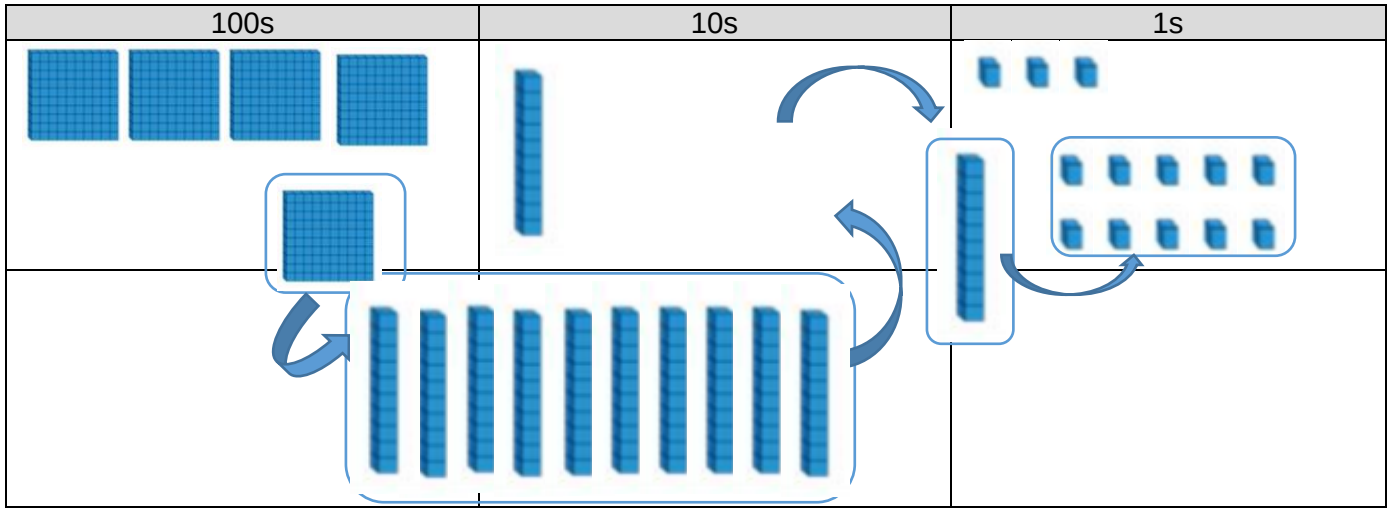
$$\begin{array}{r} 162 \\ +\,273 \\ \hline 435 \\ {\scriptstyle 1} \end{array}$$

Subtract a two-digit or three-digit number from a three-digit number, exchanging from tens and hundreds

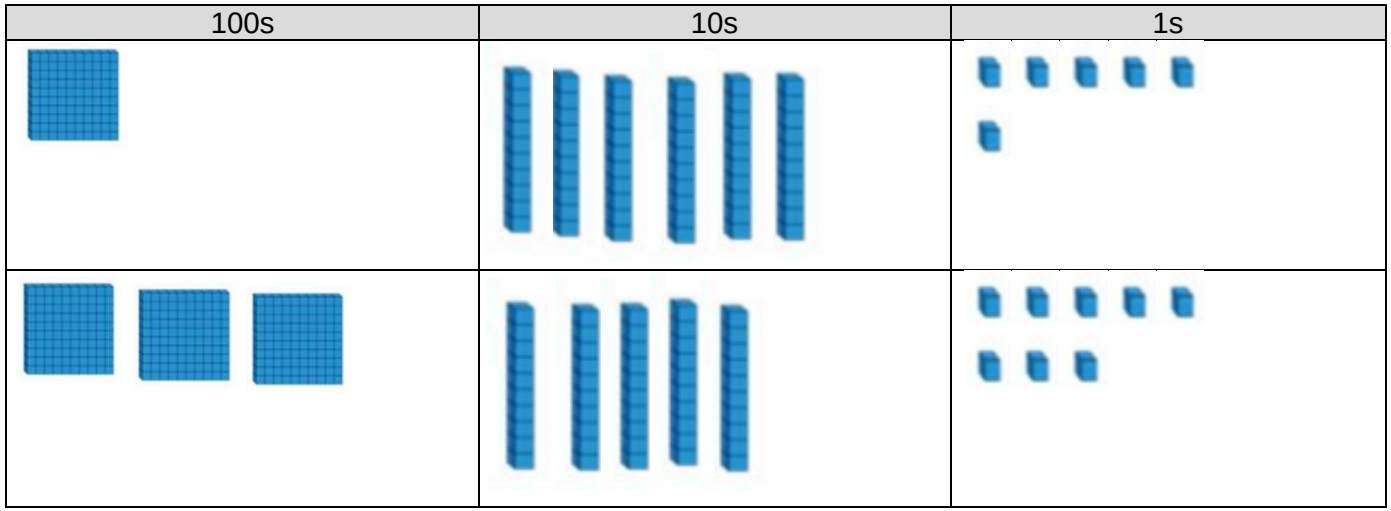
524 – 358 = 166



Children should be encouraged to reason and recognise that two exchanges are needed before calculating.



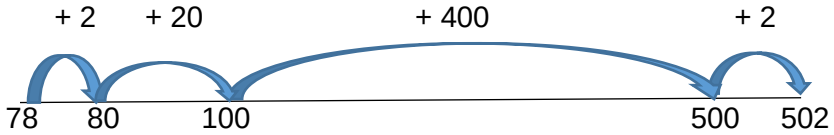
$$\begin{array}{r} {}^{4}\not{5}\,{}^{11}\not{2}\,{}^{1}4 \\ -\ 3\ 5\ 8 \\ \hline 1\ 6\ 6 \end{array}$$



$$\begin{array}{r} 1\ 6\ 6 \\ +3\ 5\ 8 \\ \hline 5\ 2\ 4 \\ \hline {\scriptstyle 1\ \ 1} \end{array}$$

Subtraction which will require exchanging through a zero

502 – 78 = 424



Children should recognise that it is actually more efficient to count on when an exchange through a zero is needed.

100s	10s	1s

One of the exchanged tens will be exchanged for ten ones.

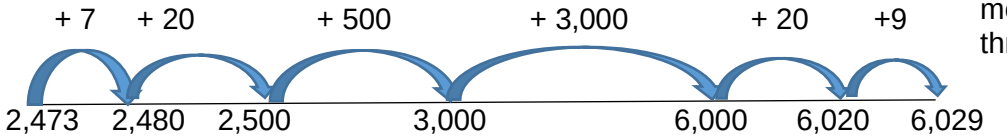
100s	10s	1s

$$\begin{array}{r} {\scriptstyle 4\ \ 9\ \ 1} \\ 5\not{0}\not{2} \\ -\ \ 7\ 8 \\ \hline 4\ 2\ 4 \end{array}$$

$$\begin{array}{r} 4\ 2\ 4 \\ +\ \ 7\ 8 \\ \hline 5\ 0\ 2 \\ {\scriptstyle 1\ \ 1} \end{array}$$

Subtract four-digit or greater numbers from a four digit number or greater, using exchanging across some or all place values, including through a zero

6,029 – 2,473 = 3,556



Children should recognise that it is actually more efficient to count on when an exchange through a zero is needed.

1,000s	100s	10s	1s
	One of the exchanged hundreds will be exchanged for ten tens.		

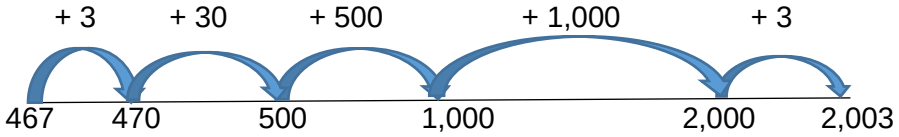
$$\begin{array}{r} {\scriptstyle 9} \\ 5\ {\scriptstyle 1}\ {\scriptstyle 1} \\ 6\not{0}\not{2}9 \\ -\,2\,4\,7\,3 \\ \hline 3\,5\,5\,6 \end{array}$$

1,000s	100s	10s	1s

$$\begin{array}{r} 3\,5\,5\,6 \\ +2\,4\,7\,3 \\ \hline 6\,0\,2\,9 \\ {\scriptstyle 1}\ {\scriptstyle 1} \end{array}$$

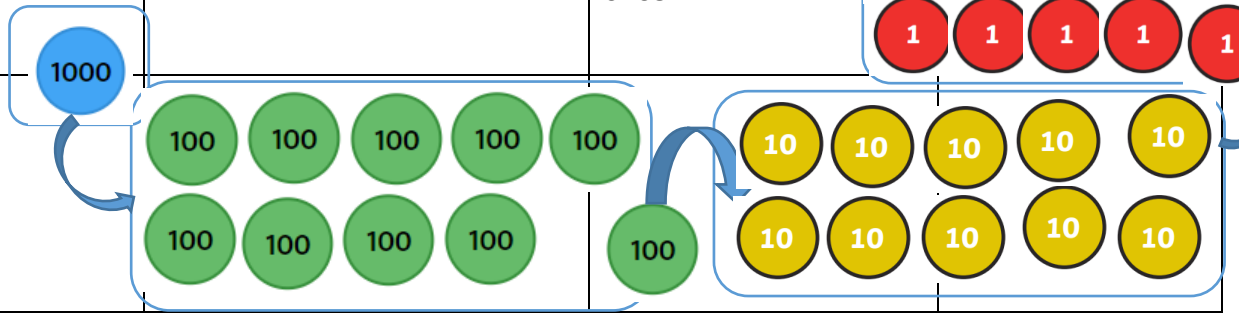
Subtract a number that will require exchanging across a series of zeros

$$2{,}003 - 467 = 1{,}536$$



Children should recognise that it is actually more efficient to count on when an exchange through a zero is needed.

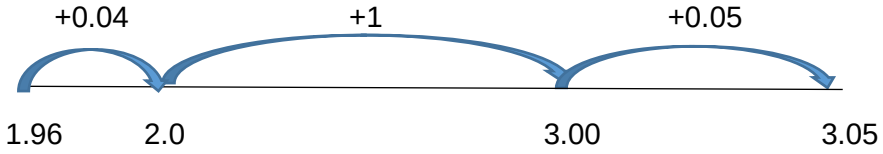
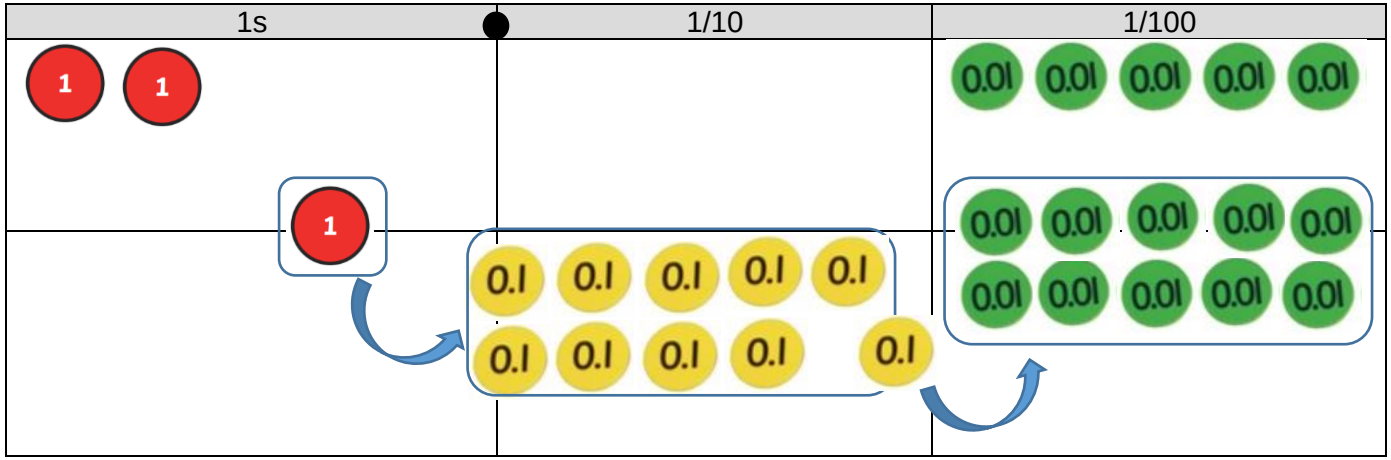
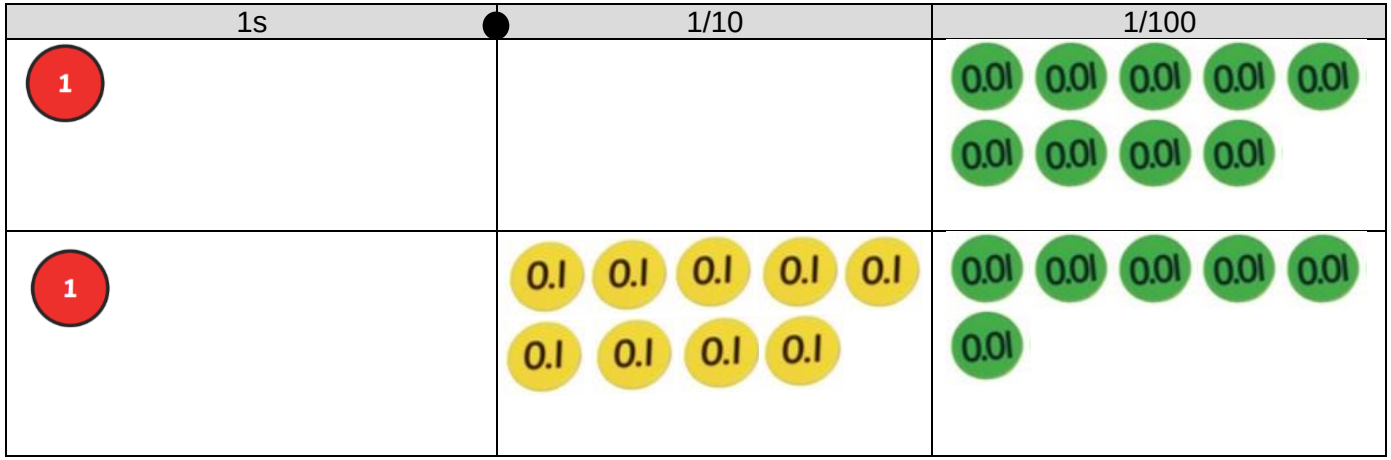
1,000s	100s	10s	1s
1000	One of the exchanged hundreds will be exchanged for ten tens.	One of the exchanged tens will be exchanged for ten ones.	



$$\begin{array}{r} {\scriptstyle 9\ 9} \\ 2\,0\,0\,3 \\ -\ \ 4\,6\,7 \\ \hline 1\,5\,3\,6 \end{array}$$

1,000s	100s	10s	1s
1000	100 100 100 100 100	10 10 10	1 1 1 1 1 1
	100 100 100 100	10 10 10 10 10 10	1 1 1 1 1 1 1

$$\begin{array}{r} 1\,5\,3\,6 \\ +\ \ 4\,6\,7 \\ \hline 2\,0\,0\,3 \\ {\scriptstyle 1\ 1\ 1} \end{array}$$

Subtract decimals, ensuring digits lined up in place values, with exchanging between decimal values and across the decimal point from ones, including through a zero.	Before using the formal method, children are to use a blank number line to become more efficient. Children should also be able to use their reasoning skills to check if their solution is reasonable.  3. 05 – 1.96 = 1.09   9 1 1 2 ~~3~~ . ~~0~~ 5 - 1 . 9 6 1 . 0 9 1. 0 9 + 1 . 9 6 3 . 0 5 1 1

Throughout all teaching of written methods for subtraction, children need to be given time to practise and consolidate skills and must be given opportunities to apply these written methods, at whatever stage they may be at to solving real-life problems, in the context of measures and money, and within the other strands of mathematics.

SUBTRACTION NATIONAL CURRICULUM CALCULATION GUIDANCE

Year 1 pupils should be taught to:

- read, write and interpret mathematical statements involving subtraction (-) and equals (=) signs
- represent and use number bonds and related subtraction facts up to 20
- subtract one-digit and two-digit numbers to 20, including zero
- solve one-step problems that involve subtraction, using concrete objects and pictorial representations, and missing number problems such as 9 - □ = 5

Year 2 pupils should be taught to:

- solve problems with subtraction using concrete objects and pictorial representations, including those involving numbers, quantities and measures
- apply their increasing knowledge of mental and written methods
- recall and use subtraction facts up to 20 fluently, and derive and use related facts up to 100
- subtract numbers using concrete objects, pictorial representations and mentally, including:
    - a two-digit number and ones
    - a two-digit number and tens
    - two two-digit numbers
- know that subtraction of one number from another cannot be done in any order
- recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems

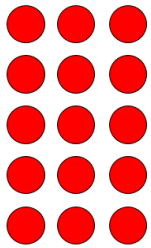
Year 3 pupils should be taught to:

- subtract numbers mentally, including:
    - a three-digit number and ones
    - a three-digit number and tens
    - a three-digit number and hundreds
- subtract numbers with up to three digits, using formal written methods of columnar subtraction
- estimate the answer to a calculation and use inverse operations to check answers
- solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction

Year 4 pupils should be taught to:
• subtract numbers with up to four digits using the formal written methods of columnar subtraction • subtract decimal numbers up to two decimal places (in the context of money and measures) • estimate and use inverse operations to check answers to a calculation • solve subtraction two-step problems in contexts, deciding which operations and methods to use and why
Year 5 pupils should be taught to:
• subtract whole numbers with more than four digits, using formal written methods of columnar subtraction • subtract decimal numbers with more than two decimal places • subtract numbers mentally with increasingly large numbers • use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy • solve subtraction multi-step problems in contexts, deciding which operations and methods to use and why
Year 6 pupils should be taught to:
• perform mental calculations, including with mixed operations and large numbers. • use their knowledge of the order of operations to carry out calculations involving the four operations • solve subtraction multi-step problems in contexts, deciding which operations and methods to use and why • solve problems involving addition, subtraction, multiplication and division • use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy

Progression in teaching multiplication:

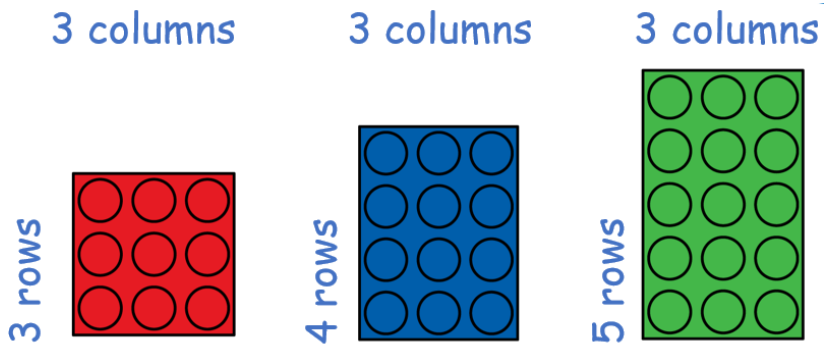
Small step/stage	Models and representations
Multiply a one-digit number by a one-digit number using arrays	Use arrays to show the relationship between arrays, multiplication and repeated addition.

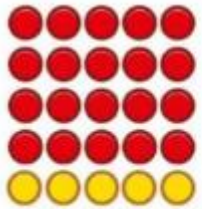


There are 5 rows.
There are 3 counters in each row.
There are 15 counters altogether.

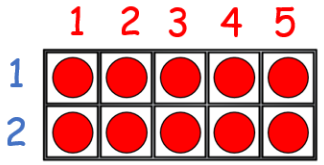
This is 2 groups of 6 and also 6 groups of 2.

Form arrays using counters to visualise commutativity.

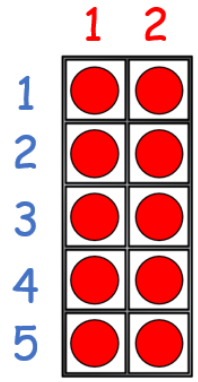




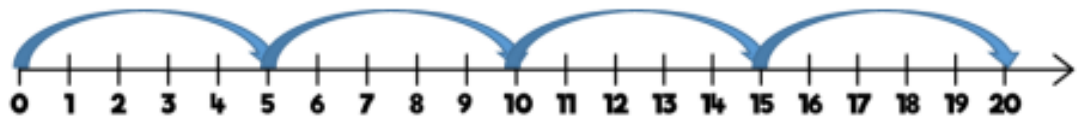
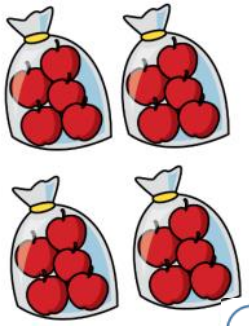
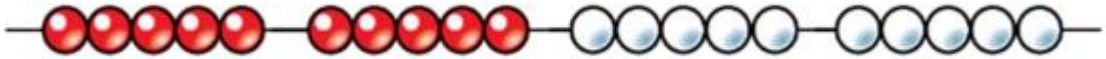
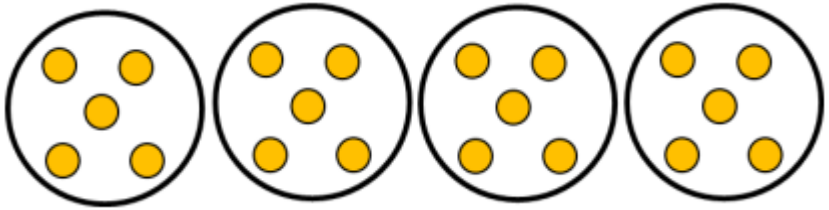
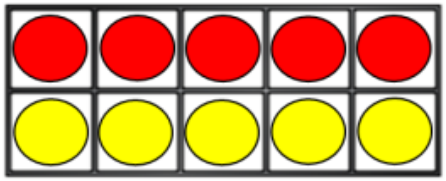
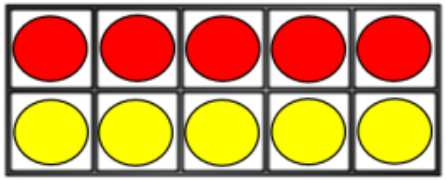
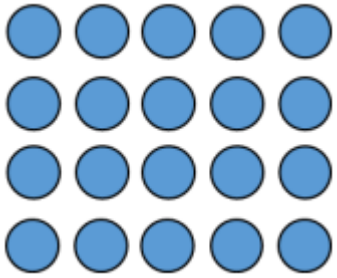
4 groups of 5 … 5 groups of 5



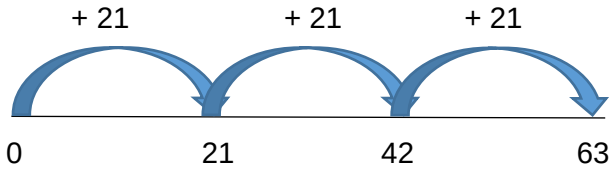
This has 2 rows.
It has 5 columns.



This has 5 rows.
It has 2 columns.

Solve one step problems using multiplication	   Children represent multiplication as repeated addition in many different ways.  One bag holds 5 apples. How many apples do 4 bags hold?  $5 + 5 + 5 + 5 = 20$ In Year 2, children are introduced to the multiplication symbol.   $5 \times 4 = 20$ 5 is the group size. 4 is what I am doing. I have 5 four times. 5 multiplied by 4 4 lots of 5 4 groups of 5 

Multiply a two-digit number by a one-digit number



21	21	21
?		

20	1	20	1	20	1
63					

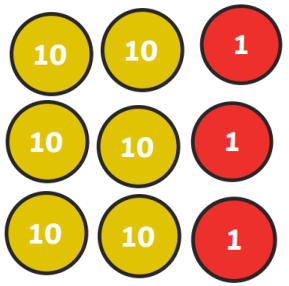
X	20	1
3	60 (2nd)	3 (1st)

Start with the bottom right calculation.

$$\begin{array}{r} 21 \\ \times\ 3 \\ \hline 63 \end{array}$$

$$\begin{array}{r} 21 \\ \times\ 3 \\ \hline 3 \quad (1 \times 3) \\ +\ 60 \quad (20 \times 3) \\ \hline 63 \end{array}$$

The grid method should be a short step between the number line and concrete representations to the expanded method.



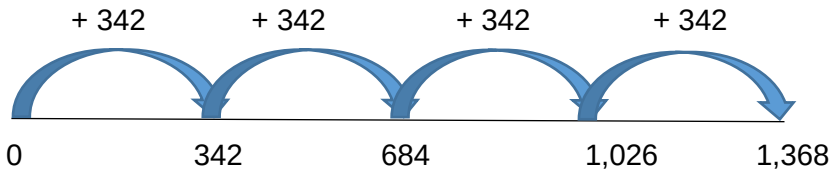
10s	1s
10 10	1
10 10	1
10 10	1

When I multiply 21 by 3 the product is 63.

21 is the group.

21 x 3 = 63
21 three times
21 multiplied by 3

Multiply a three-digit number by a one-digit number



342	342	342	342
?			

300	40	2	300	40	2	300	40	2	300	40	2
1,368											

X	300	40	2
4	1,200 (3rd)	160 (2nd)	8 (1st)

3 4 2
X 4
8 (2 x 4)
1 6 0 (40 x 4)
\+ 1 2 0 0 (300 x 4)
1 3 6 8

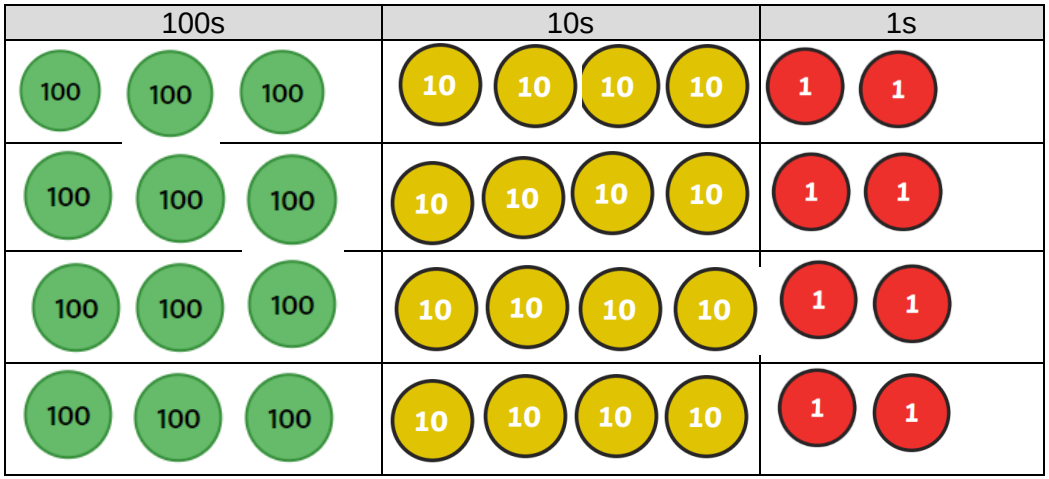
The grid method should be a short step between the number line and concrete representations to the expanded method.

Start with the bottom right calculation.

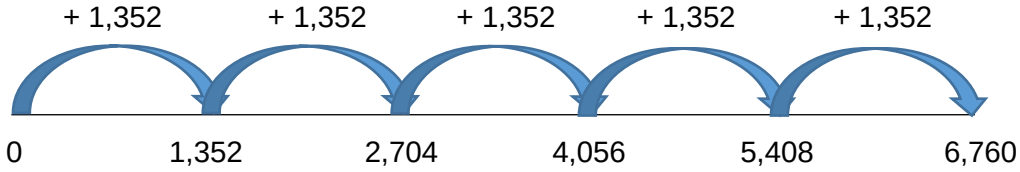
3 4 2
X 4
1 3 6 8
1

When I multiply 342 by 4 the product is 1,368.

342 is the group.



Multiply a four-digit number by a one digit number



1,352	1,352	1,352	1,352	1,352
?				

X	1,000	300	50	2
5	5,000 (4^{th})	1,500 (3^{rd})	250 (2^{nd})	10 (1^{st})

Start with the bottom right calculation.

When I multiply 1,352 by 5 the product is 6,760.

1, 352 is the group.

```
  1 3 5 2
X       5
-------
  6 7 6 0
  1 2 1
```

```
  1 3 5 2
X       5
-------
    1 0  (2 x 5)
  2 5 0  (50 x 5)
1 5 0 0  (300 x 5)
+ 5 0 0 0 (1000 x 5)
-------
  6 7 6 0
```

1,000s	100s	10s	1s
1000	100 100 100	10 10 10 10 10	1 1
1000	100 100 100	10 10 10 10 10	1 1
1000	100 100 100	10 10 10 10 10	1 1
1000	100 100 100	10 10 10 10 10	1 1
1000	100 100 100	10 10 10 10 10	1 1

Multiply a two-digit number by a two-digit number

X	20	2
30	600 (4th)	60 (3rd)
1	20 (2nd)	2 (1st)

Start with the bottom right calculation.

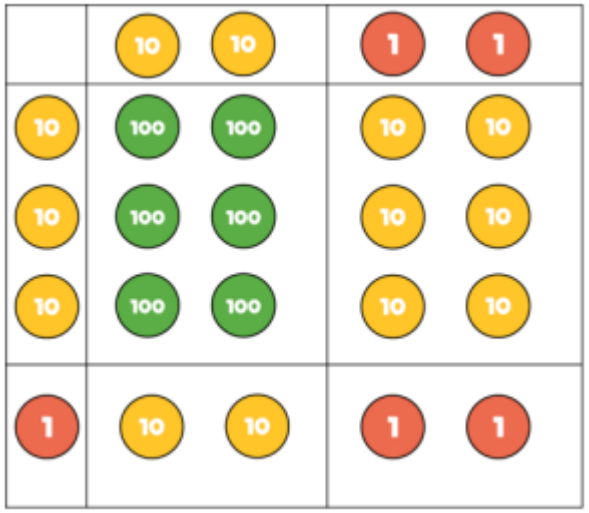
```
  2 2
X 3 1
    2 (2x 1)
  2 0 (20 x 1)
  6 0 (2 x 30)
+ 6 0 0 (20 x 30)
  6 8 2
```

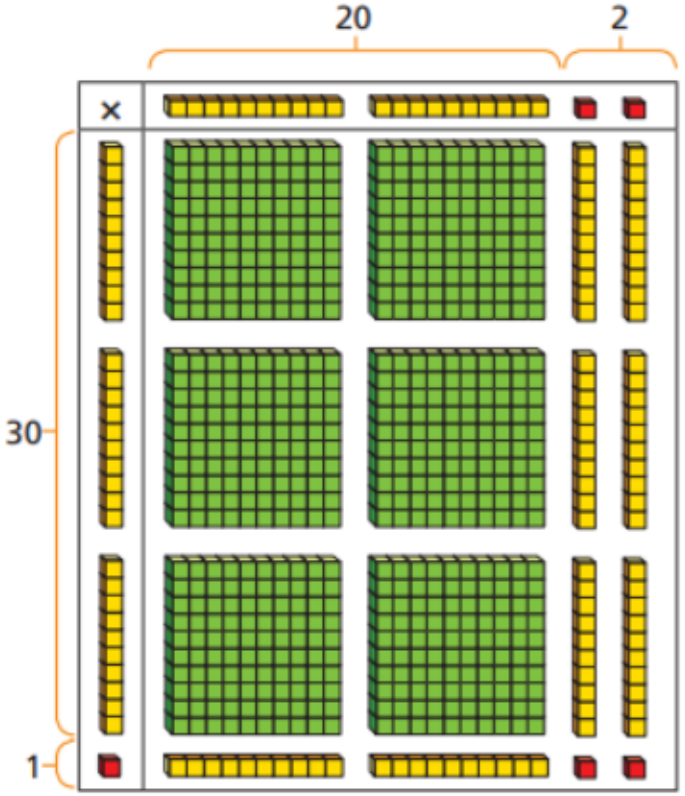
```
  2 2
X 3 1
  2 2
+ 6 6 0
  6 8 2
```

When I multiply 22 by 31 the product is 682.

22 is the group.

The area model will help children understand the size of the numbers they are using.





Multiply a two-digit number by a three- digit number

X	200	30	4
30	6,000 (6th)	900 (5th)	120 (4th)
2	400 (3rd)	60 (2nd)	8 (1st)

Start with the bottom right calculation.

When I multiply 234 by 32 the product is 682.

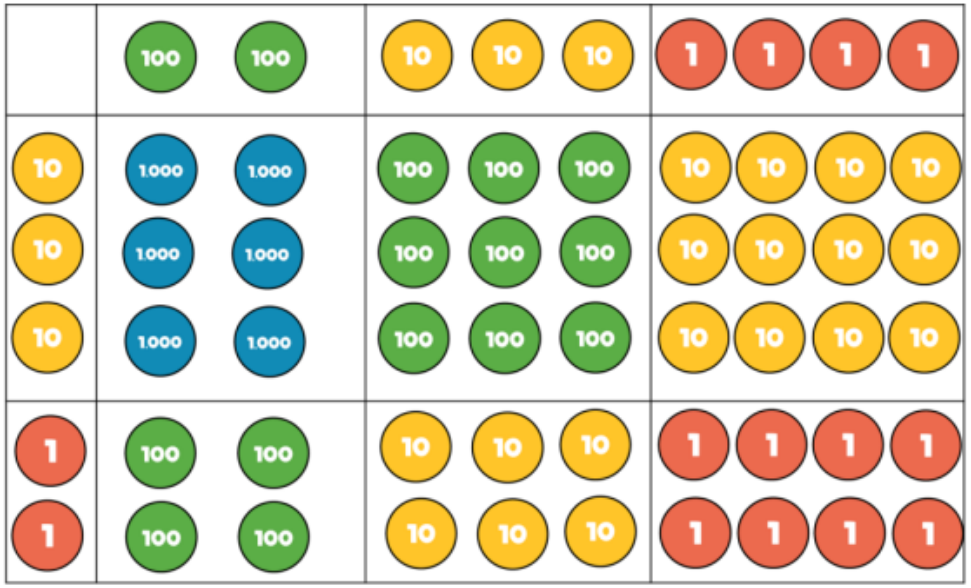
234 is the group.

The area model will help children understand the size of the numbers they are using. When multiplying a three-digit number it is more practical to use counters.

```
  2 3 4
  X 3 2
  -----
      8  (4 x 2)
    6 0  (30 x 2)
  4 0 0  (200 x 2)
  1 2 0  (4 x 30)
  9 0 0  (30 x 30)
+6 0 0 0 (200 x 30)
  -------
  7 4 8 8
  1 1
```

```
  2 3 4
X   3 2
  -----
    4 6 8
+ 7 0 2 0
  -------
  1 1
  7 4 8 8
```

```
  2 3 4
X   3 2
  -----
    4 6 8
+ 7 0 2 0
  -------
  7 4 8 8
  1 1
```



Multiply a two-digit by a four-digit number

X	2,000	700	30	9
20	40,000 (8th)	14,000 (7th)	600 (6th)	180 (5th)
8	16,000 (4th)	5,600 (3rd)	240 (2nd)	72 (1st)

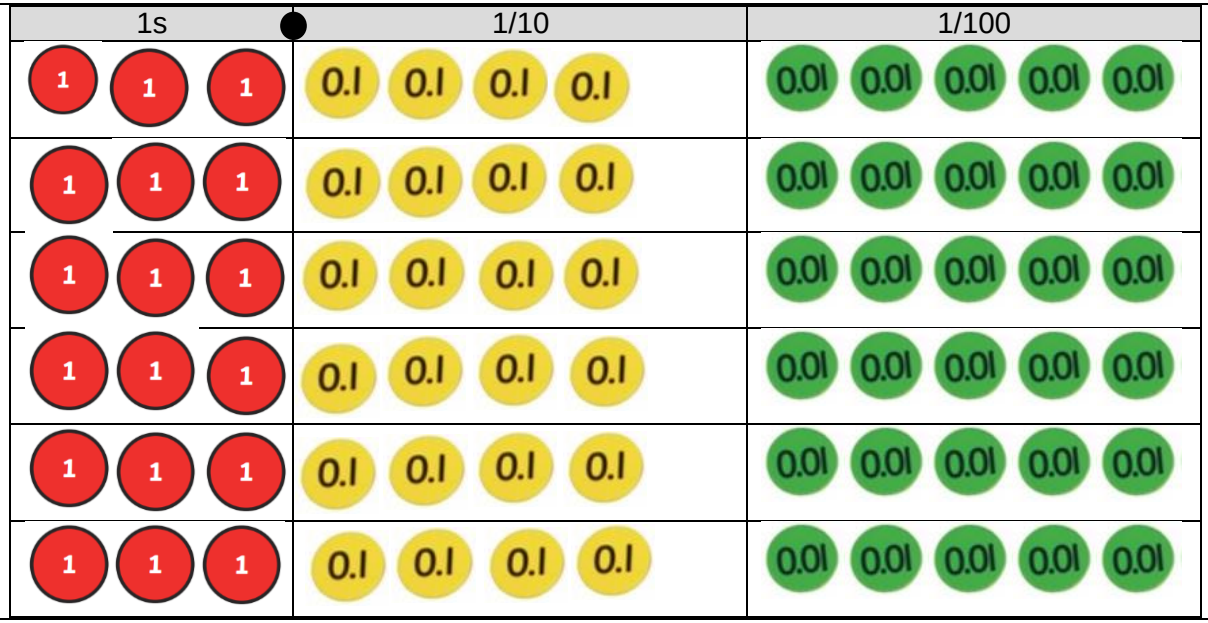
When I multiply 2, 732 by 28 the product is 76, 692.

2, 732 is the group.

```
  2 7 3 9
X     2 8
-------------
    7 2   (9 x 8)
  2 4 0   (30 x 8)
  5 6 0 0 (700 x 8)
1 6 0 0 0 (2,000 x 8)
    1 8 0 (9 x 20)
    6 0 0 (30 x 20)
  1 4 0 0 0 (700 x 20)
+ 4 0 0 0 0 (2,000 x 20)
-------------
  7 6 6 9 2
  1 1 1
```

```
  2 7 3 9
X     2 8
-------------
  2 1 9 1 2
  2 5 3 7
+ 5 4 7 8 0
  1   1
-------------
  7 6 6 9 2
```

Multiply decimal numbers up to three decimal places




```
  3 . 4 5
X 6
-------------
  1 8       (3 x 6)
    2 . 4   (0.4 x 6)
+   0 . 3   (0.05 x 6)
-------------
  2 0 . 7
```

```
  3 . 4 5
X 6
-------------
  2 0 . 7
  1
```

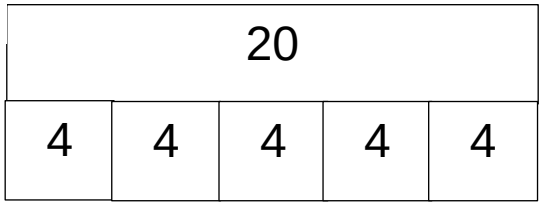
MULTIPLICATION NATIONAL CURRICULUM CALCULATION GUIDANCE

Year 1 pupils should be taught to:
• solve one-step problems involving multiplication, calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher
Year 2 pupils should be taught to:
• recall and use multiplication facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers • calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication (×) and equals (=) signs • show that multiplication of two numbers can be done in any order (commutative) • solve problems involving multiplication using materials, arrays, repeated addition, mental methods, and multiplication including problems in contexts
Year 3 pupils should be taught to:
• recall and use multiplication facts for the 3, 4 and 8 multiplication tables • write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written method • solve problems, including missing number problems, involving multiplication including integer scaling problems and correspondence problems in which *n* objects are connected to *m* objects (ratio and proportion)
Year 4 pupils should be taught to:
• recall multiplication facts for multiplication tables up to 12 × 12 • use place value, known and derived facts to multiply mentally, including: multiplying by 0 and 1; multiplying together three numbers • recognise and use factor pairs and commutativity in mental calculations • multiply two-digit and three-digit numbers by a one-digit number using formal written layout • solve problems involving multiplying and adding, including using the distributive law to multiply two-digit numbers by one digit, integer scaling problems and harder correspondence problems such as *n* objects are connected to *m* objects (ratio and proportion)
Year 5 pupils should be taught to:
• solve problems involving multiplication where larger numbers are used by decomposing them into their factors • multiply numbers up to four-digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers • multiply numbers mentally drawing upon known facts • multiply whole numbers and those involving decimals by 10, 100 and 1000 • solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign • solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates • solve problems involving multiplication, including using their knowledge of factors and multiples, squares and cubes
Year 6 pupils should be taught to:

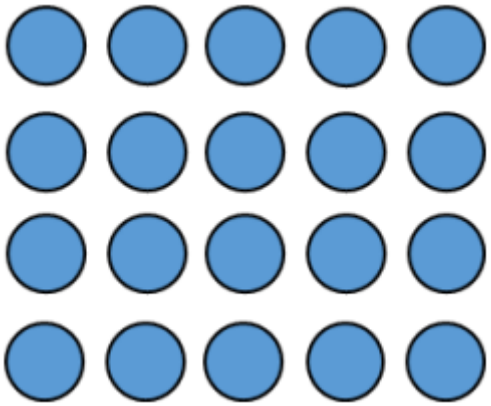
- multiply multi-digit numbers up to four-digits by a two-digit whole number using the formal written method of long multiplication
- multiply decimal numbers up to three decimal places
- perform mental calculations, including with mixed operations and large numbers.
- use their knowledge of the order of operations to carry out calculations involving the four operations
- solve multi-step problems involving addition, subtraction, multiplication and division
- use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy

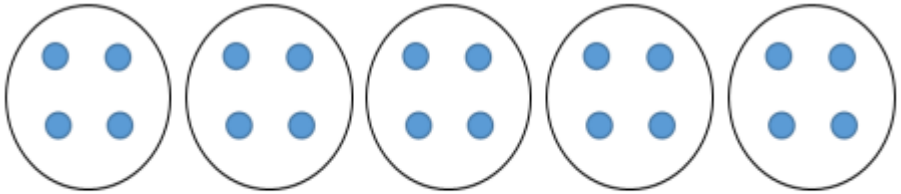
Progression in teaching division:

Small step/stage	Models and representations
Solve one step problems using multiplication (sharing)	Children solve problems by sharing amounts into equal groups.



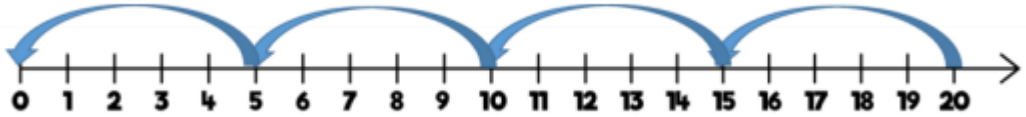
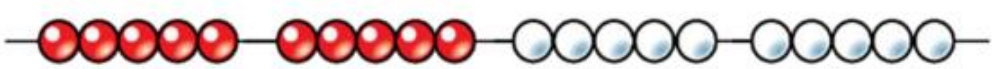
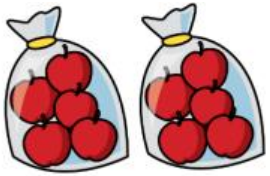
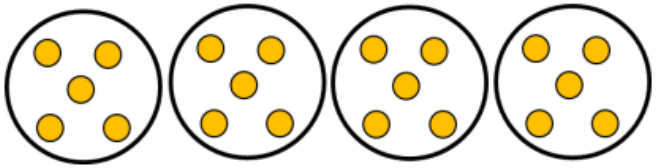
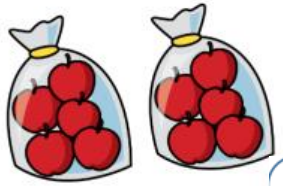
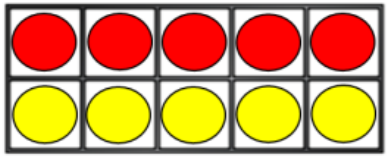
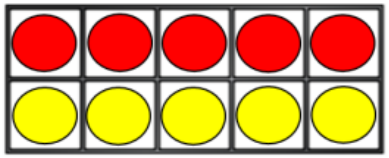
There are 20 apples altogether.
They are shared equally between 5 bags.
How many apples are in each bag?





$20 \div 5 = 4$

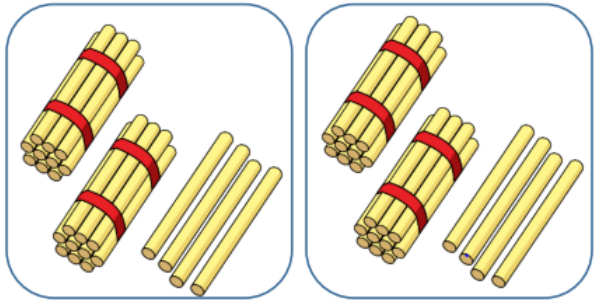
5 is the group size. How many groups of 5 are there?

Solve one step problems using division (grouping)	        There are 20 apples altogether. They are put in bags of 5. How many bags are there?

Children solve problems by grouping and counting the number of groups. Grouping encourages children to count in multiples and links to repeated subtraction on a number line. They can use concrete representations in fixed groups such as number shapes which helps to show the link between multiplication and division.

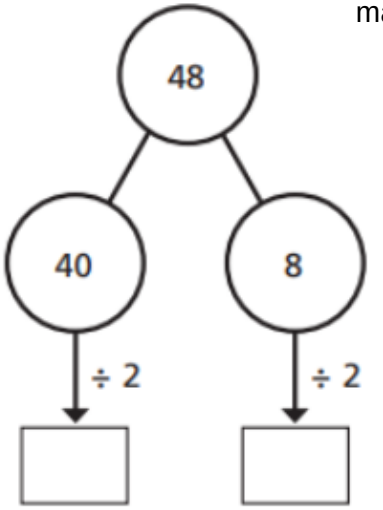
Divide a two-digit number by a one-digit number (no exchange)	

Tens	Ones
10 10	1 1 1 1
10 10	1 1 1 1

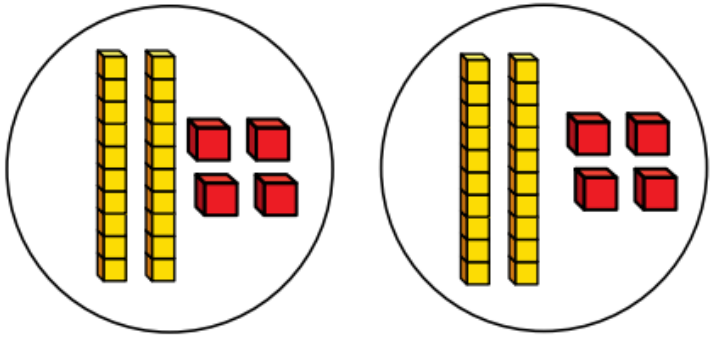


Children need to use manipulatives that will allow them to partition into tens and ones.

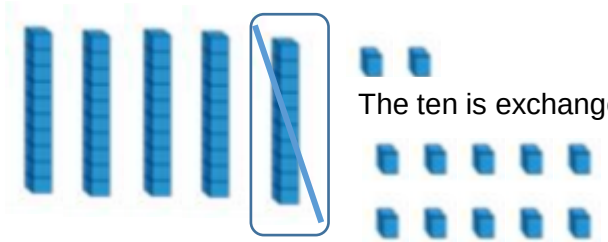
Part-whole models can provide children with a clear written method that matches the concrete representation.



48 ÷ 2 = 24

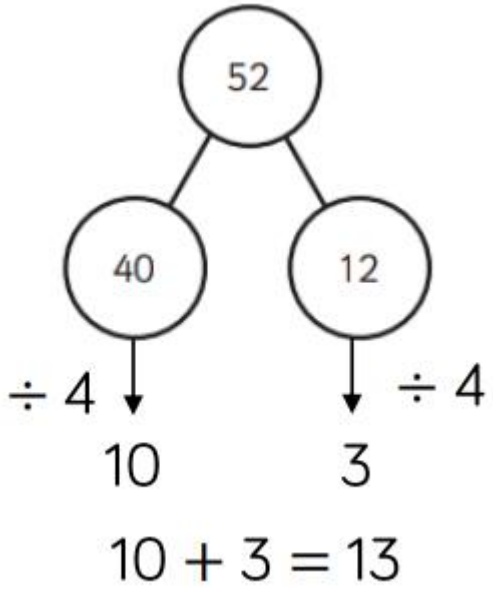


Divide a two-digit number by a one-digit number (with exchange)



The ten is exchanged for ten ones.

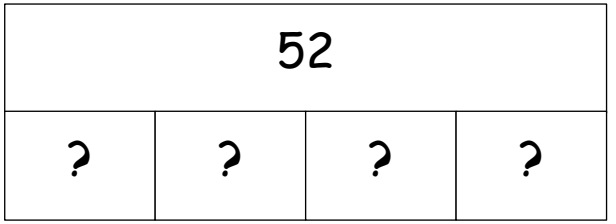
10s	1s



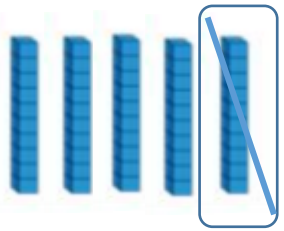
When dividing numbers involving an exchange, children can use dienes and place value counters to exchange one ten for ten ones.

Children should start with the equipment outside the place value grid before sharing the tens and ones equally between the rows.

Flexible partitioning in a part-whole model to be drawn next to the representation so that children can make the link (important for long division).

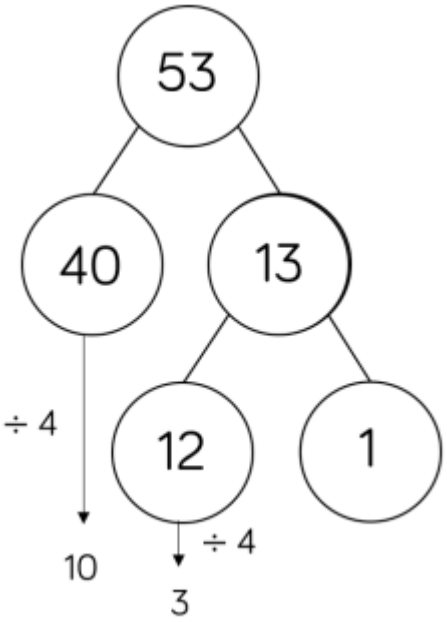


Divide a two-digit number by a one-digit number with a remainder (sharing)



The ten is exchanged for ten ones.

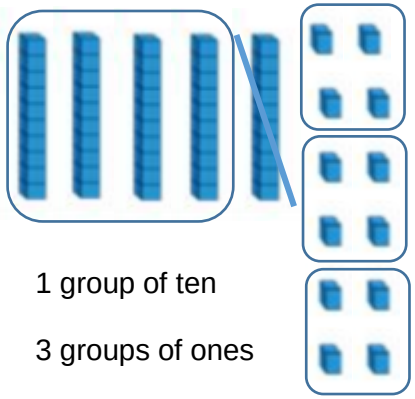
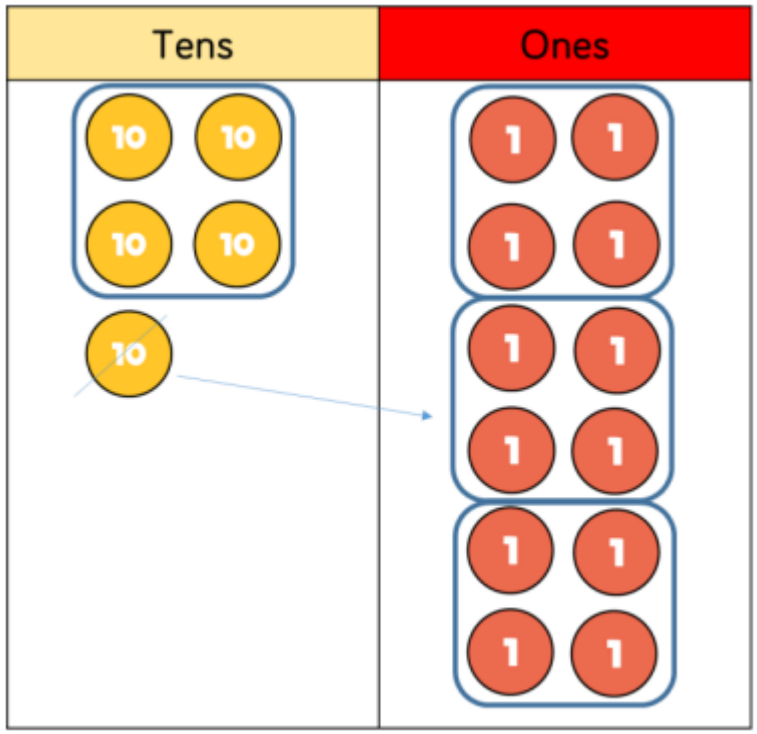
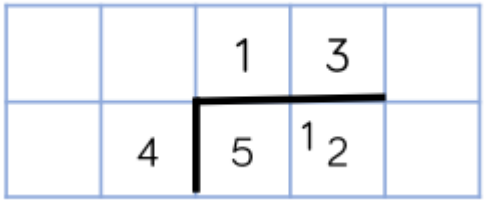
10s	1s



Starting with the equipment outside the place value grid will highlight remainders as they will be left outside the grid once the equal groups have been made. Encourage children to use knowledge of times tables e.g. all multiples of 4 are even so 53 is not divisible by 4.

Again, flexible partitioning in a part-whole model to be drawn next to the representation so that children can make the link (important for long division).

53				
13	13	13	13	1

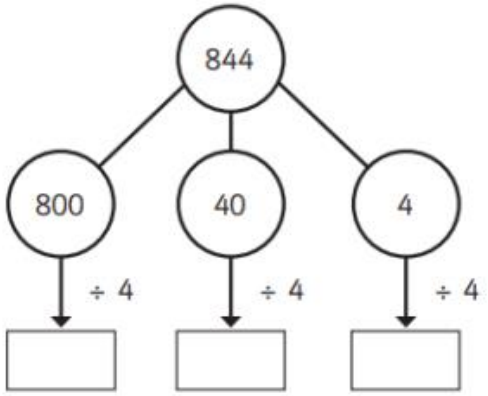
Divide a two-digit number by a one-digit number with a remainder (grouping)	$52 \div 4 = 13$  1 group of ten 3 groups of ones   When using the short division method, children use grouping. Starting with the largest place value, they group by the divisor. Language is important here. Children should consider 'How many groups of 4 tens can we make? And 'How many groups of 4 ones can we make?' Remainders can also be seen as they are left ungrouped.

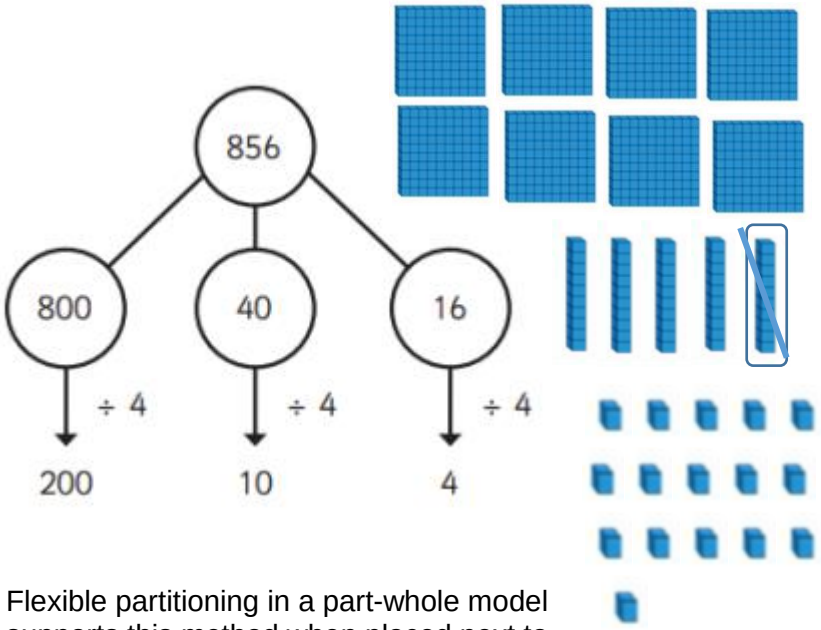
Divide a three-digit number or greater by a one-digit number (sharing)

844 ÷ 4 = 211

H	T	O
100 100	10	1
100 100	10	1
100 100	10	1
100 100	10	1

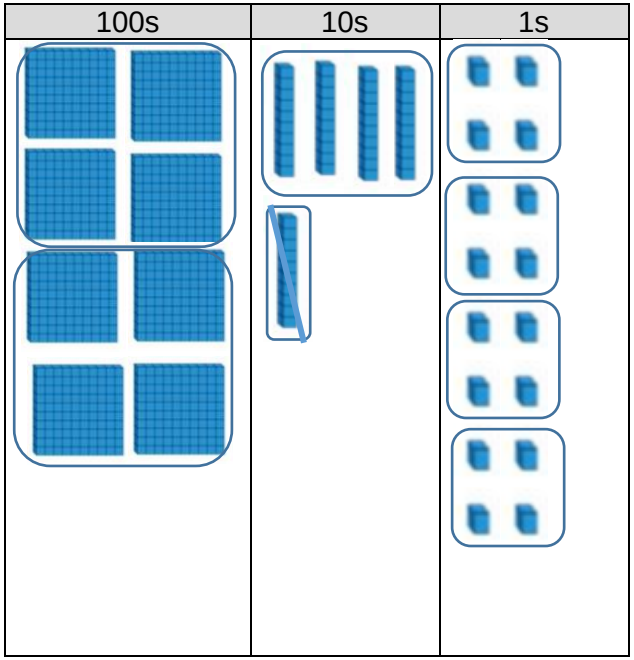
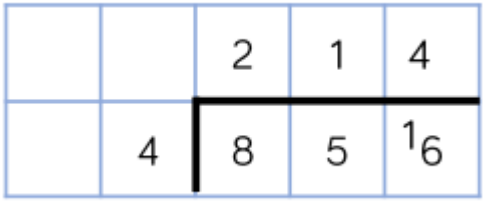
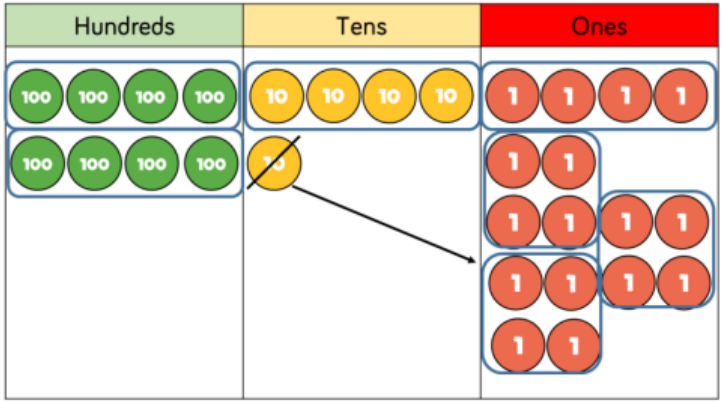
Children can continue to use place value counters to share 3-digit numbers into equal groups.

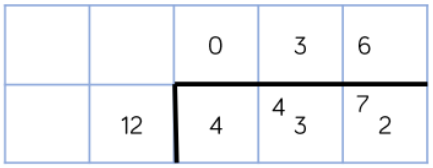


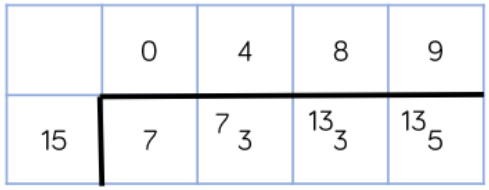


Flexible partitioning in a part-whole model supports this method when placed next to the concrete. Children can see how 56 is partitioned into 40 and 16.

100s	10s	1s

Divide a three-digit number or greater by a one-digit number (grouping)	 856 ÷ 4 = 214   Grouping will continue to support children's understanding of short division when dividing a three-digit number by a one-digit number. Dienes and place value counters can be used or drawn on a place value grid to support.

<table>
<tr>
<td>Divide a three-digit number or greater by a two-digit number, including with a remainder (short division)</td>
<td>




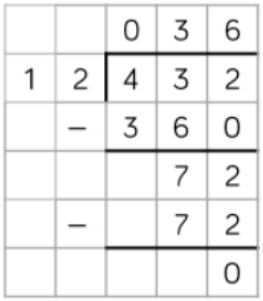
7335 ÷ 15 = 489

432 ÷ 12 = 36

15 x 1 = 15
15 x 2 = 30
15 x 3 = 45
15 x 4 = 60
15 x 5 = 75
15 x 6 = 90
15 x 7 = 105
15 x 8 = 120
15 x 9 = 135
15 x 10 = 150

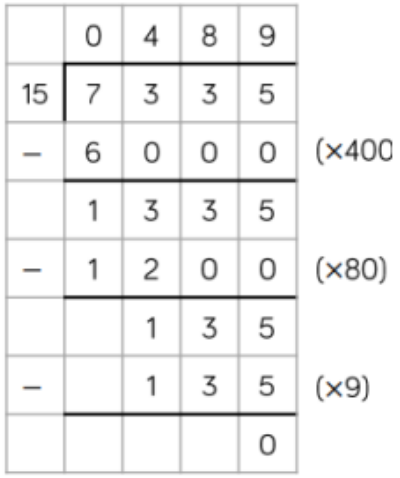
When children begin to divide up to four-digits by 2-digits written methods become the most accurate.

Children can write out multiples to support their calculations with larger remainders.

Children will also solve problems with remainders where the quotient can be rounded as appropriate.
</td>
</tr>
<tr>
<td>Divide a three-digit number or greater by a two-digit number, including with a remainder (long division)</td>
<td>


(×30)
(×6)

12 × 1 = 12
12 × 2 = 24
12 × 3 = 36
12 × 4 = 48
12 × 5 = 60
12 × 6 = 72
12 × 7 = 84
12 × 8 = 96
12 × 7 = 108
12 × 10 = 120



(×400
(×80)
(×9)

1 × 15 = 15
2 × 15 = 30
3 × 15 = 45
4 × 15 = 60
5 × 15 = 75
10 × 15 = 150
</td>
</tr>
</table>

Divide a decimal number by a one-digit or two-digit number, including with a remainder	If children are dividing a decimal by an integer, the short division method can be used. $0.744 \div 6 = 0.124$ $\quad$ $87.5 \div 7 = 12.5$ $$\begin{array}{r} 0.124 \\ 6\overline{)0.744} \\ {\scriptstyle 1\ 2} \end{array} \qquad \begin{array}{r} 12.5 \\ 7\overline{)87.5} \\ {\scriptstyle 1\ 3} \end{array}$$
Express remainders as fractions and decimals of divisor	$$\begin{array}{cccccccc} & & & 2 & 4 & r & 1 & 2 \\ 1 & 5 & 3 & 7 & 2 & & & \\ & - & 3 & 0 & 0 & & & \\ & & & 7 & 2 & & & \\ & - & & 6 & 0 & & & \\ & & & 1 & 2 & & & \end{array}$$ $1 \times 15 = 15$ $2 \times 15 = 30$ $3 \times 15 = 45$ $4 \times 15 = 60$ $5 \times 15 = 75$ $10 \times 15 = 150$ $$\begin{array}{cccccc} & & & 2 & 4 & \frac{4}{5} \\ 1 & 5 & 3 & 7 & 2 & \\ & - & 3 & 0 & 0 & \\ & & & 7 & 2 & \\ & - & & 6 & 0 & \\ & & & 1 & 2 & \end{array}$$ $$\begin{array}{r} 035{\cdot}5 \\ 4\overline{)142{\cdot}0} \\ {\scriptstyle 1\ 2\ 2\ 2} \end{array}$$

Throughout all teaching of written methods for division, children need to be given time to practise and consolidate skills and must be given opportunities to apply these written methods, at whatever stage they may be at to solving real-life problems, in the context of measures and money, **including where in real-life contexts the answer needs to be rounded**.

DIVISION NATIONAL CURRICULUM CALCULATION GUIDANCE

Year 1 pupils should be taught to:
• solve one-step problems involving division, calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher
Year 2 pupils should be taught to:
• recall and use division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers • calculate mathematical statements for division within the multiplication tables and write them using the division (÷) and equals (=) signs • show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot • solve problems involving division, using materials, arrays, repeated addition, mental methods and multiplication and division facts, including problems in contexts
Year 3 pupils should be taught to:
• recall and use division facts for the 3, 4 and 8 multiplication tables • write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods • use the numberline to divide, using the principle of repeated subtraction • solve problems, including missing number problems, involving division, including integer scaling problems and correspondence problems in which *n* objects are connected to *m* objects (ratio and proportion)
Year 4 pupils should be taught to:
• recall division facts for multiplication tables up to 12 × 12 • use place value, known and derived facts to divide mentally, including dividing by 1 • use the numberline to divide, subtracting chunks of numbers to make the method more efficient • develop chunking method through moving away from the numberline to record vertically • recognise and use factor pairs and commutativity in mental calculations

Year 5 pupils should be taught to:

- divide numbers mentally drawing upon known facts
- divide numbers up to four-digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- divide whole numbers and those involving decimals by 10, 100 and 1000
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates
- solve problems involving multiplication, including using their knowledge of factors and multiples, squares and cubes

Year 6 pupils should be taught to:

- divide numbers up to four-digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- divide decimal numbers up to three decimal places
- perform mental calculations, including with mixed operations and large numbers.
- use their knowledge of the order of operations to carry out calculations involving the four operations
- solve problems involving addition, subtraction, multiplication and division
- use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy

Vocabulary

Addition

sum, add, altogether, increase, more than, total

We do not say answer - we say sum.

Subtraction

difference, subtract, take, minus, less than, decrease, reduce

We do not say answer - we say difference.

Multiplication

times, multiplied by, groups of, factor

We do not say answer - we say product (the result of multiplying one number by another)

Factor – a number that multiplies with another to make a product.
3 x 4 = 12
3 and 4 are factors
12 is the product

5 x 4 = 20
5 is the group size.
4 times is what I am doing.

I have 5 four times.
5 multiplied by 4
4 lots of 5
4 groups of 5

Division

divided by

We do not say answer - we say quotient (the result of a division)

Divisor – the number by which another is divided
Dividend – the number that is divided

24 ÷ 6 = 4
6 is the divisor
24 is the dividend
4 is the quotient

Other vocabulary:

Zero

Negative numbers

We only say minus when learning about temperature

We say improper fraction – not top heavy

We do not say regroup or borrow, we say exchange.
Exchange – change a number or expression for another of the same value

Calculation/number sentence/expression (2+4=6)

We say 'equal' 'the same as' 'is equal to'
20 = 9 + 11